UdTU Assignment 4A+B - Case: DCAMM Ph.D. course “Wind Turbine Dynamics and Aeroelasticity”

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UdTU Assignment 4A+B - Case: DCAMM Ph.D. course “Wind Turbine Dynamics and Aeroelasticity”

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Abstract (max. 2000 char.):

This report contains the answers to Assignment 4A and 4B in the course "Education in University Teaching at DTU (UdTU)" at the Learning Lab DTU. The answers are based on a case study built on a new Ph.D. course held in the period 23rd – 27th of June, 2008. It is first of all a documentation of the experiences and suggestions for improvements that came out of giving this course. It will help me prepare for the next run of the course, or maybe help my colleagues that want to teach a similar course, or take over this one. The writing of the assignment has also helped me reflect more on my experiences, and many of the suggested improvements may not have been brought up without this reflection. Writing a similar, but shorter, document for each of my courses may be a good idea, especially for the analysis of the student feedback from the course evaluation questionnaires. It could help structure and remember the course improvements. Finally, this assignment has helped me reflect on the collection of knowledge obtained from all the UdTU modules. Writing down this reflection may also help me remember what I have learned.

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Contents

Preface

This report contains answers to Assignment 4A and 4B in the course "Education in University Teaching at DTU (UdTU)" at the Learning Lab DTU. The answers are based on a case study built on a new Ph.D. course held in the period 23rd – 27th of June, 2008.

The UdTU course consists of four modules:

- Teaching and Learning (Module 1)
- Teaching Methods and Planning (Module 2)
- Feedback and Evaluation (Module 3)
- Teaching Development project (Module 4)

where Module 4 consists of the present case study based on an actual teaching situation (for more information see http://www.learninglab.dtu.dk).

I would like to thank my colleague, Bjarne S. Kallesøe, for his assistance during the course, especially with the supervision of group work with the exercises.

1 Assignment 4A

The topics of Assignment 4A are an introduction and analysis of the case study course, discussions of the pre-test of the students and preliminary impressions from the planned course start, and finally a presentation of an overall question that will be given special attention throughout this case study.

1.1 Course introduction

The case study course is a 2.5 ETCS point Ph.D. course at the research school of the Danish Center for Applied Mathematics and Mechanics (DCAMM). Hence, there is no official DTU course description; however, the flyer that announced the course is attached as an appendix to this report. This flyer has an almost fixed format including a brief course description, where the learning objectives for the course have been added in the present flyer as a novelty. The typical and present flyers do not include a clear description of the prerequisites, probably because it is expected that Ph.D. students are able themselves to evaluate if they are qualified; an assumption will show to be wrong in this case.

Under the section title "Participants", the flyer states: "*This course is aimed at graduate students interested in wind energy research and engineers working in the wind turbine industry with purpose that they learn how to describe, analyze, and optimize the modal dynamics and aeroelastic stability of wind turbines based on the commercial three-bladed concept. The course participants are presumed to have pre-knowledge of vibration analysis and linear dynamics of mechanical systems. No particular knowledge of wind turbines is needed; however, some general knowledge of structural dynamics and unsteady aerodynamics is presumed*". To somebody who knows the topics of wind turbine dynamics and aeroelasticity, these vague formulations of the prerequisites may be clear, but not for others, as discussed later in this report.

There were fifteen Ph.D. students and fifteen industrial engineers in the course, see picture below. Ph.D. students do not pay a registration fee, but employees in academia and industry pay 250 and 750 euro, respectively. To the industry, this registration fee is quite low, and knowing the wind energy industry it is clear that many of the industrial participants did not have any pre-knowledge and were send on the course for basic training in wind turbine dynamics.

1.2 Course objectives and challenges

The levels of understanding in a Ph.D. course should be quite high on the scale of Bloom's taxonomy as indicated by the words "analyze" and "optimize" in the course flyer. However, when writing the learning objectives into the flyer, the levels was lowered to accommodate for an expected high number of industrial participants and the limited time available for reaching higher levels of understanding of the complex wind turbine dynamics.

1.2.1 Learning objectives

The students are supposed to develop the competences described by the following learning objectives:

- <u>describe</u> the methods of computing the modal frequencies, damping, and shapes of the structural and aeroelastic modes of operating wind turbines, and <u>apply</u> these methods for turbine analysis.
- <u>describe</u> the modal dynamics of three-bladed wind turbines, and to <u>identify</u> these structural modes in the aeroelastic response of operating wind turbines.
- <u>describe</u> the mechanisms of the aeroelastic instability called stall-induced vibrations, and <u>explain</u> which turbine parameters that determine the stability limits with respect to stall-induced vibrations.
- <u>describe</u> the mechanisms of the aeroelastic instability called classical flutter, and <u>explain</u> which turbine parameters that determine the flutter limits.
- <u>work</u> with state-of-the-art topics within the area of dynamics and aeroelastic stability of wind turbines.

These learning objectives are set up to cover the core elements of the course, as presented in the following. All objectives lie on the lower levels *Knowledge*, *Comprehension*, and *Application* of Bloom's taxonomy. As mentioned, these levels are chosen to fit with the participants' pre-knowledge, and the limited time for reaching higher levels for the topic of wind turbine modal dynamics and stability on which the first scientific papers were published in the early 2000's. The last learning objective is vaguely formulated with the purpose to suggest the participants, especially the Ph.D. students, to continue their work within the subject, and thereby reach higher levels of understanding. They learn about the methods and phenomena here to explore them further in their own research.

1.2.2 Core elements

The learning objectives are set up to cover five different groups of core elements:

- Terminology of wind turbine dynamics and aeroelasticity:

    - These core elements concern terminology used within the wind energy community, which are the basis for communicating other knowledge in the course. For example the different turbine concepts (what is stall- and pitch-regulated turbines?), the names of the dominating degrees of freedom of turbines (what are tilt, yaw, and azimuth angles?), and the names of turbine modes. Without the knowledge of these terms, the students will not be able to learn about the other core elements.

- Methods for performing modal and aeroelastic stability analyzes on wind turbines:

    - These core elements concern the special methods for performing modal and aeroelastic stability analyzes on wind turbines with a rotating rotor leading to periodic coefficients in the equations of motion. It is assumed that the participants, as a prerequisite, can describe and apply traditional methods within linear vibration analysis of mechanical systems without a rotating part (e.g. a turbine at standstill) based on eigenvalue analysis of time-invariant equations of motion. A core element in this group of core elements is a method based on the use of a coordinate transformation (the so-called *Coleman transformation* into *multi-blade coordinates*) to eliminate the periodic coefficients in the equations of motion and thereby enabling the use of eigenvalue analysis. A connected core element is the knowledge of the limitations of this method. Other methods for performing linear vibration analysis of the periodic systems, e.g. Floquet theory, are mentioned in the course, but they do not (yet) qualify as a core element because they are given to little attention.

- Modal dynamics of three-bladed wind turbines:

    - These core elements concern the modal dynamics of three-bladed wind turbines that is derived from applying the above method of Coleman transformation followed by eigenvalue analysis. They contain answers to questions like: what happens to the modes of a standstill turbine when the rotor is rotating? Why do some modes couple into whirling modes? What happens to the modal frequencies? Why do some modes lead to multiple resonance frequencies in the rotating frame of reference? Answers to all these and related questions are core elements that represents knowledge needed for the students to learn about the aeroelastic instabilities of wind turbines, which belongs to the next group of core elements.

- Aeroelastic instabilities of three-bladed wind turbines:

    - These core elements concern the two aeroelastic instabilities that have, or are likely to occur on wind turbines, namely stall-induced vibrations and classical flutter. The understanding of the mechanisms and important parameters behind the fluid-structure interactions that lead to these instabilities represent the final knowledge towards which the other core elements are pointing: wind turbine terminology and methods for eigenvalue analysis is basis for describing the modal dynamics of wind turbines, and this underlying dynamics and its interaction with the surrounding flow determine the aeroelastic stability limits of wind turbines. The ability to understand this connection between modal dynamics and aeroelasticity is an important element in the design of wind turbines.

- State-of-the-art in wind turbine dynamics and aeroelasticity:
    - This core element is included because an ambition with the course is to create a larger interest for the above core elements in academia and industry. Knowledge of the present state-of-the-art is the starting point of any further development, or research by the students.

Interestingly, new core elements may develop during the course because it concerns subjects under current research. The results of the students' group work, their questions, and the discussions with them is likely to uncover open issues and new research topics that qualify as new core elements (e.g. the use of Floquet theory) that must be incorporated into the next run of the course.

1.2.3 Difficulties and teaching challenges

It is expected that the students will have difficulties with the theoretical background and mathematics of the methods used for analysis, and with the physical comprehension of the sometimes counter-intuitive modal dynamics of wind turbines. For example, it is assumed by the prerequisites that the students know what a mode is, and how eigenvalue analysis can be used to compute the modal properties. In retrospect, this assumption is not valid. But even if these basic concepts are clear to the students, the modal dynamics of turbines with a rotating rotor is sometimes so counter-intuitive that different research papers disagree on the subject.

The main challenge for the teacher in this course is to teach a complex subject to students with very different backgrounds. Participants from the industry may have forgotten how the learning process works, and focus very much on solving the problem and less on understanding the background of the methods to do so. They can be categorized as Competence and Job oriented students (cf. UdTU Module 2), and they spend most of their time in the part of Kolb's learning cycle dealing with Active Experimentation and Concrete Experience (cf. UdTU Module 2). The Ph.D. students on the other hand are often Science oriented students, and they spend most of their time on Reflective Observation and Abstract Conceptualisation in Kolb's learning cycle. For them a practical problem may be secondary, whereas the methods and their theoretical implications are more interesting for them. The challenge is to force all course participants through all parts of Kolb's learning cycle.

1.3 Course framework and methods

The ideas behind the planning of the course were to have one intensive week of combined dialog-based lectures, group work with exercises, and student presentations and discussion of own results. These ideas were based on the assumption of relative few participants, with a tentative maximum of 20. The combination of dialog-based lectures, exercises, and student presentations was planned to make participants from the industry go through the reflective and abstract parts of Kolb's learning cycle by letting them present (and defend) their results to other students that may ask reflective and abstract questions out of their own preferences. On the other, the exercises were planned to make Ph.D. students become active in their learning by using the methods and obtaining concrete experiences.

1.3.1 Course outline, teaching methods and material

The tentative course outline presented in the flyer ended up being very close to the actual one, which can be found in the course notes appended to this report. The first four days of the course week are very similar: each morning begins with breakfast, followed by three hours of discussion of yesterday's exercise (except for Monday), lectures on theory, and introduction of that afternoon's exercise. These four days take place in a class room during the mornings and in a databar during the afternoons. On Friday, the course is moved to Risø – DTU, where the morning is similar to the other days with breakfast and discussion of yesterday's exercise, but without a lecture on new theory. The Friday afternoon is devoted for a small excursion and course evaluation.

The teaching methods contained in this outline can be divided into three qualitatively different groups:

- **Class room lectures**

  The wind turbine terminology, methods for modal and stability analyses and the mechanisms of aeroelastic instabilities for wind turbines are taught in dialog-based lectures using slides as foundation and small buzz group questions to activate the students. An example: Before any introduction of wind turbine terminology, a slide with animations of the first eight modes of a turbine at standstill is shown to the students, and they are asked to name and order them after their frequency. Some students name the modes based on prior knowledge, whereas others without prior knowledge of wind turbines come up with their own names. All names are written on the whiteboard, and it is very clear that a common terminology is needed. The ordering of the modes after frequencies follows a similar pattern; students with prior knowledge are challenged by the other students to explain their ordering and vice versa. This dialog takes place in the buzz groups and in the class room when the right ordering is put together through dialog with the teacher. Such dialogs during the lectures sometimes have an inductive character, where the problem (e.g. ordering of modes) is analyzed based on intuition and conception which is well-developed in mechanical engineers.

- **Group work with exercises**

  Hands-on experiences with modal and stability analyses of wind turbines are the objective of the group work in the afternoons, where the students must program their own code that can compute the modal and stability properties of a simplified wind turbine using the methods that are taught in the mornings. The first section of the exercise description states: "*During this course you should imagine that you are working in a company that has started the design of a wind turbine where the preliminary simulations show large problems with vibrations. You must identify the source of these vibrations and suggest design improvements using the methods introduced in the course*". Their solutions to these problems are based on their code and their understanding of how to use it. The exercises for each afternoon are described in a document which is appended to this report. Ten groups (each named after a color) of three persons are formed so that there are Ph.D. students and participants from the industry in each. The discussion and collaboration in these mixed groups are assumed to help each group member through the whole circle of Kolb's learning cycle by doing the programming and analyzing/validating the results.

- **Student presentations**

  The discussion of yesterday's results during morning lectures begins with a quantitative comparison by the teacher of all groups' results by plotting the numbers in diagrams, where the line colors shows which group has computed the particular results. Groups that stand out are allowed to explain their deviations. To have the students reflect on their results and experiences from the exercises, they are asked to present how their results help analyze the vibration problem of the test turbine. All groups cannot present each day due to time limitations, but two to three groups are asked each afternoon to present some of their results the next day.

The teaching material consists of

- Course notes that were send out to the participants a week before the course to introduce its content and to provide the reading material for preparation that consists of three papers covering all topics of the course.
- Exercise notes containing a description of the exercise turbine, the assignments and a derivation of the simplified aeroelastic model used for the analyses.
- Slides prepared for the classroom lectures and the slides prepared by the students for their presentations.
- A demo version of the software HAWCStab that can be used to do the same analyses as required in the exercise. This demo is provided at the end of the course.

All these documents and the HAWCStab demo are uploaded to the web-site http://teamsites.risoe.dk/dcamm-wtda from which the student can download them after the course.

1.3.2 Assessment methods

The course flyer states: "*To pass the course, active participation and work with assignments are required*". This vague formulation is inherited from the common practice used in previous one-week Ph.D. courses at the DCAMM research school. The short time available for the teaching leaves little time for evaluation; however, the students are also given a "post-test" to evaluate the overall learning outcome. This post-test is an exact copy of the anonymous pre-test (discussed below), and it will therefore not be possible to identify learning of each student by this post-test.

The "active participation" of each student is evaluated by the teachers especially during the exercises in the afternoon by letting each of the three group members ask and answer questions, and during the presentations of each group's results. The active students quickly standout by the enthusiasm and curiosity, and the focus is therefore put on the less active students.

The common practice in one-week Ph.D. course of requiring "only" active participation as assessment of the student learning is not ideal, nor fair if students do not pass, because the requirement is so vaguely formulated. But the very limited time for teaching and learning does not allow for allocation of extra time for assessment during the course week. A solution is the use of written assignments that are done by the students after the course.

1.3.3 Possible improvements

The final number of 30 participants changed an original plan of two students per group for the exercise to three students, because the number of groups should not be too high (more than ten), otherwise some groups would not be able to present a result from the exercise. There will always be problems with an inactive "third-wheel" in three-person groups, which requires extra attention.

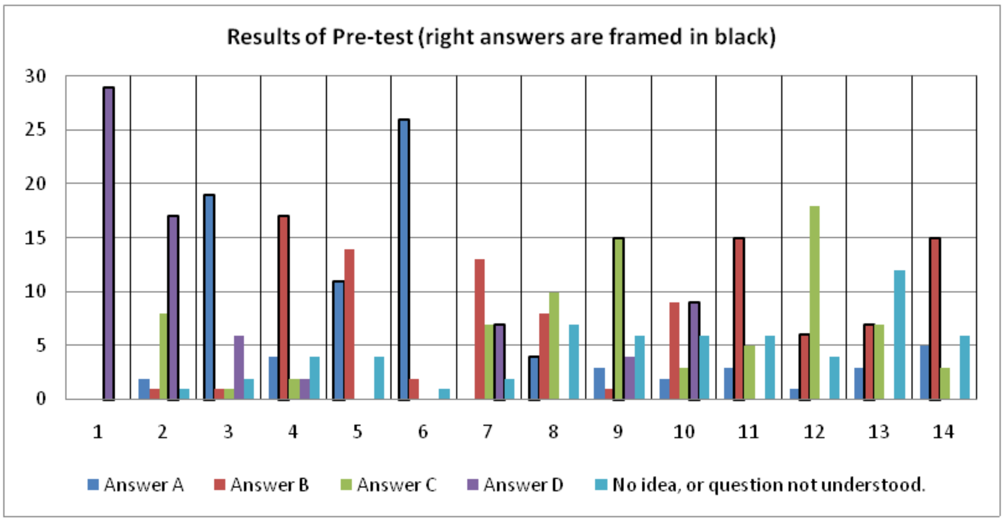
The time for preparing the slides and the classroom lectures for a new course like the present was underestimated. A lot of the preparation time was used on the designing the exercise so that the students would have to handle all the core elements during the group work. This priority was intentional, but more time for preparation will be an improvement.

1.4 Pre-test of students

To assess the students' prerequisites, they were given a multiple-choice test consisting of fourteen questions that were displayed in an automated slide show[1], where each slide was displayed for one minute. The test is designed as a test of both the prerequisites and the learning outcome; some questions will therefore cover the core elements of the course and are not supposed to be answered correctly in the pre-test. To avoid "false right answers" due to statistics of pure guessing, the students are giving the possibility to answer: "No idea, or question not understood".

The first four questions concerns wind turbine terminology, which should all be answered correctly by someone studying or working with wind turbine engineering. Questions 5 – 7 concern modal dynamics of wind turbines at standstill, which could be answered correctly by students that study or work daily with wind turbine dynamics. Question 8 concerns modal dynamics of wind turbines in operation, which requires that the student has extensive knowledge of wind turbine dynamics; a trial of the pre-test showed that colleagues came up with wrong answers. Questions 9 – 14 concern aeroelastic stability of wind turbines, which require that the student has extensive knowledge of wind turbine aeroelasticity. These questions are together with Question 8 aimed at the core elements.

The diagram below shows the answers of 29 students (one student was missing during the first hours of the course) to the fourteen questions with up to four possible answers (A, B, C, or D) and the "no idea"-answer. The right answer to each question is marked by adding a black frame around the corresponding answer-bar.



[1] The slide show is appended to this report. It contains videos of wind turbine mode shapes that show up as black boxes on the print-out, but the PDF file is opened in a newer Adobe Reader, they may viewed individual by clicking each of them. Note that in the slide show the video is looped so that the mode shapes are animated continuously.

All students answered the first question correctly, which shows that they know something about the terms used for the different degrees of freedom of a wind turbine. Over half of the students answered questions 2 – 4 correctly, which shows that they also know something about the operational conditions of wind turbines. Question 5 is a 50/50 question, where most students were wrong, which indicate that they cannot quantify the modal frequencies of typical turbines. However, the many right answers to Question 6 shows that they have a qualitative conception of the mode shape naming, but again the few right answers to Question 7 they cannot quantify their modal frequencies relative to each other. The many "no idea" answers to Question 8 were expected; those students who had an idea about the right answer (or just guessed), may have read some of the pre-course reading material, where they can find the right answer. There are many right answers to Questions 9 – 14, which that some students either have a good intuition about aeroelastic instabilities of wind turbines, or they have read about/work with these issues before. Note that there are many "no idea" answers to Questions 8 – 14 that cover the core elements of the course.

These results of the pre-test were deduced Monday evening. They were as expected, and did not implicate the need of changes in the course planning, or teaching methods. However, the pre-test lacks a test of some prerequisites as discussed in the following description of the first impression.

1.5 First impressions

All practical issues e.g. registration of the students Monday morning in the class room, getting breakfast from the cantina, access to computers in the databar, etc. worked perfectly. The students were all very motivated and there was a nice atmosphere with small talk around the breakfast and in the coffee breaks.

The first lectures Monday morning on went fine with good interaction with the students in dialogs, especially the small buzz-groups discussion seemed to help the dialog. There was one problem with a student that kept interrupting me and the other students when they were answering a question. He did it clearly out of eagerness, and it helped a little when I asked him in private to let others speak, but at some point I had to introduce "hand-raising before speaking" to avoid having him answering all questions. Hand-raising does not improve a free dialog.

Just before lunch on Monday, the exercise was introduced and the practical issues with computer access for each group were cleared. After lunch, the groups quickly got started on that day's exercise, which involved eigenvalue analysis for a single wind turbine blade based on the equations of motion that were given to the students in a MatLab file. Linear vibration analysis is a main prerequisite for the course, and this simple analysis was not assumed to be a problem for the students. However, it quickly became clear that some students did not know that the modes of a mechanical system are determined from the eigensolutions of an eigenvalue problem derived from its equations of motion. All groups were asked to come to the lecture room for a brief introduction of these basics of linear vibration analysis on the white board. It solved some of the misunderstandings but it was clear that such lecture should have been given in the morning. Furthermore, the pre-test was not designed for detecting this missing prerequisite, because it was (naively) assumed that the students had understood what was meant by the prerequisites described in the course flyer. During the introduction of eigenvalue analysis at the whiteboard, a participant from the industry claimed that it was the first time he had seen this theory and that he did not know what prerequisites were required for attending the course. This comment symbolized that the flyer needs rewriting to clearly list the theoretical backgrounds needed for attending the course. All groups went back to the databar and worked until (and some beyond) scheduled ended of the first day. My colleague and I had a bad feeling that the level was set too high, and that the students would not be able to go through the whole exercise (from which they should "learn by doing").

1.6 An overall question

My overall question that I will give special attention during the course is: How do I teach a mixed group of Ph.D. students and industrial engineers to make sure that they all reach the learning objectives? As really mentioned, the choice of class-room teaching with dialog and buzz-groups combined with a practical exercise is aimed at handling the differences between research and problem oriented students.

2 Assignment 4B

Assignment 4B concerns a review of the course from the points of view of both teacher and students: Did the course plan work? Did the teaching methods have the intended effect? Did the students reach the learning objectives? How did the students evaluate the course? What can be changed to improve the course? Finally, the use and effect of the peer-coaching is also reviewed, and the overall question of how to teach a mixed group of research and industrial students is discussed.

2.1 Course deviations

The course did not deviate significantly in form from the originals plans; however, the overestimation of the students' pre-knowledge about linear vibration analysis, which showed clearly during the first exercise Monday afternoon, led to changes in the use and ambition of the exercise. The discussions during Tuesday and Wednesday mornings of their results of the exercises from Monday and Tuesday afternoons did not include presentations by the students, but were only based on the quantitative comparisons of their computed results. Seven groups delivered their results of Monday's exercise, whereas nine groups managed to deliver their results after Tuesday's exercise. It seemed that the students accelerated their learning curves after the first day, maybe because the participants from the industry needed a while to get into a "study mode" again, or because they became more familiar with eigenvalue analysis which forms the basis for all exercises. The same nine groups delivered their results of the exercises on Wednesday and Thursday, and five of these groups presented their interpretations and conclusions of these results with respect to the vibration problems of the test turbine. An example of the slides from such presentation is given in Appendix E.

The group that did not deliver any results was working hard during the exercises; however, they had chosen to write their own MatLab codes instead of using those pieces of code and suggestions for "quick'n'dirty" implementations that were given to them. Their complex programming approach caused them to be a day behind with the exercise, so that they each day finished the exercise from the previous day.

2.2 Effect of teaching methods

The class-room lectures worked well at times with a dialog, e.g. when using buzz-groups and when the students asked questions. However, the slides that were given low-priority in the busy days of preparing the course did not always help the learning process. Many slides were generalizing and meant to give an overview. The interests of some students were dropping whenever a slide/topic was too far from the framework of the exercise. It was clear that the intention of having the exercise as the centre of learning worked very well, but it also seemed to limit the possibility of giving some students an overview in this short course. The slides dealing with the theory behind the methods used in the exercise were not going into depth with the underlying mathematics, because it would steal time and attention away from the usage of the methods. As discussed later, some students (presumably the research oriented Ph.D. students) wished this theory had been presented with full derivations of the methods.

Most students, both Ph.D. students and participants from the industry, seemed to learn best during the exercise by asking questions, discussing mistakes, and explaining their results to each other and the teachers. Theory and physical interpretations of the results with respect to the vibration problem of the test turbine were mixed in a natural and fruitful way. Often, the same questions came up in all groups and the most efficient way would have been the answer them in plenum; however, our answers improved and the students reflected more on the answers by having the individual discussions in each group. Besides being part of the student assessment, these discussions with the groups also formed the basis for asking some groups to present their results and conclusions to the other groups.

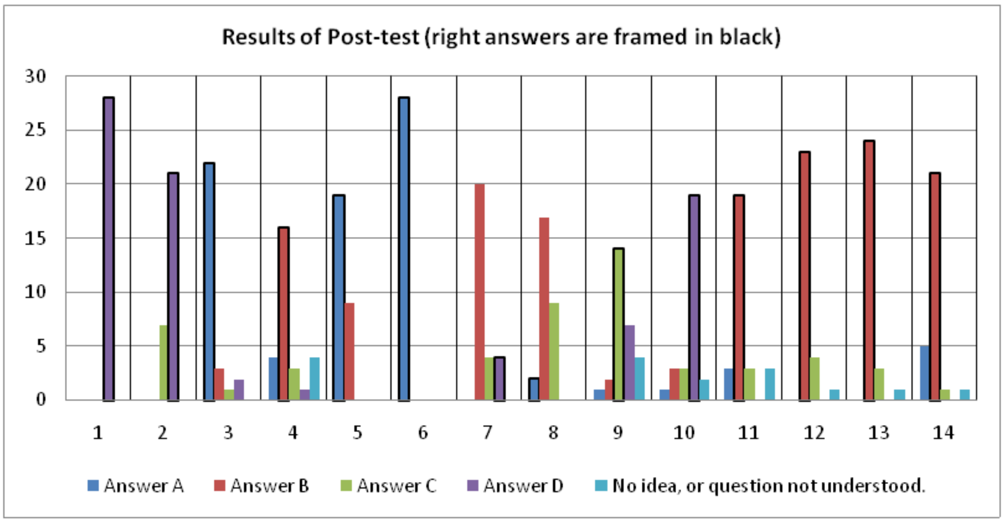
These student presentations seemed more effective for the reflection and conception of the presenting students than for helping their audience through the same parts of Kolb's learning cycle, as expected because the latter are less active. Ideally, all groups should therefore have had the opportunity to do such presentations.

2.3 Discussion of learning assessment and learning outcome

The assessment using the discussions with the groups only gives an indication of the "strong" and "weak" students. It cannot quantify the learning outcome of each student, and can only be used for grading "pass". It is difficult to fail a student based on these discussions, because it is not possible to clearly explain to the student why he/she should fail. There is no formal exam in such Ph.D. course, and all students passed the present course.

There was one person, where there were some doubts about his learning outcome, because he isolated himself from the group and worked alone on trying to understand the underlying mathematics of the introduced methods, rather than trying them out in practice. Unfortunate, he was part of the group with the over-complicated programming approach, which may also have been the reason for his isolation. We asked the group to work together and adopt the quick'n'dirty programming approach, but another group member insisted on continuing their (his) approach.

As mentioned, the students were given the same anonymous multiple-choice test as used in the pre-test at the end of the course to assess the overall learning outcome (without being able to assess the individual learning). The test results are shown in this diagram (only 28 answers due to two absent students):



Except for Questions 7 – 9, there has been a significant improvement in the test results, which indicate that the teaching of the core elements related to Questions 5, 6, and 10 – 14 has been successful. The majority of students seem to have reached the learning objectives regarding “Aeroelastic instabilities of three-bladed wind turbines” covered by Questions 9 – 14. The students are therefore assumed to have obtained a conceptual understanding of the aeroelastic stability characteristics of commercial wind turbines and their determining design parameters. The reduced score on Question 9 dealing with stall-induced vibrations may be explained by a misunderstanding of the question, as seen by the high number of “no idea” answers, and this question must be rephrased.

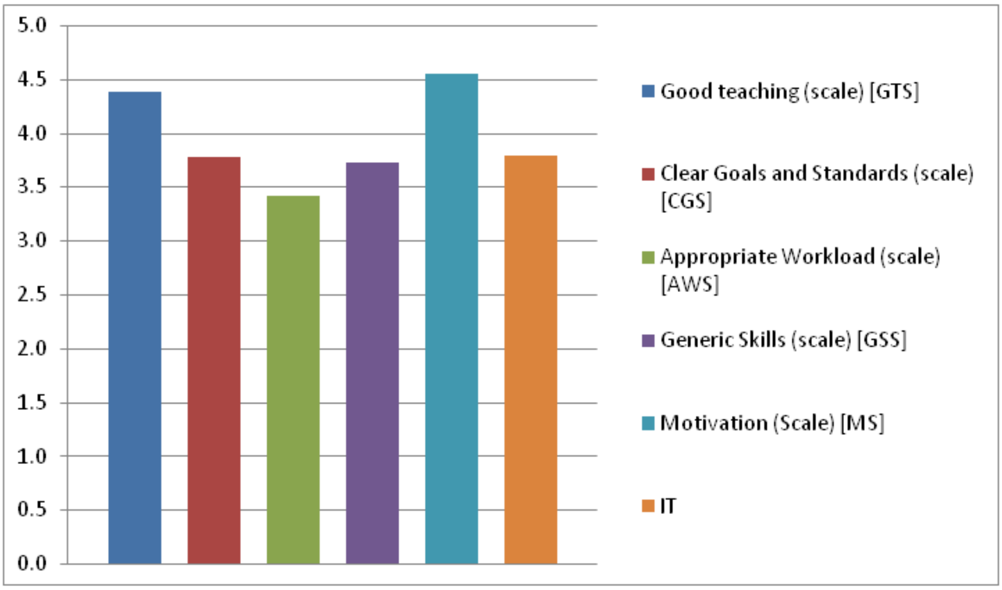
The low score on Question 7 shows that the students still had problems ordering the modal frequencies of wind turbines at standstill, which is one of the core elements regarding “Modal dynamics of three-bladed wind turbines”. The question is a little tricky because two of the turbine modes are related to the same blade mode and therefore have modal frequencies close to each other, but the right ordering has been covered in both a lecture and in the exercises. Similar, the low score on Question 8 can be explained by misunderstanding/-reading of the question. It is a trick question that requires a good conceptual understanding of the modal dynamics of three-bladed wind turbines in operation. The colleagues trying the test before the course explained that they had not seen (read) the last part of the question: “*… when measured on the blade*”. They suggested that I should have highlighted this part to avoid their misunderstanding; however, I felt that I would reveal the answer by doing that … it would become too easy. When this change was discussed with the students after their post-test, they all agreed that I should have done as my colleagues suggested. The problem is that the majority of their answers showed that they had read that last part of the question, because they gave the answer where a frequency measured in the fixed frame is added and subtracted the rotor speed to give the frequency measured on the rotating blade, except that they had not read that the frequency f in the question refers to a blade frequency. In conclusion, the trick question ended up being a source of a good discussion afterwards, which could be included more directly in the course, if an alternative post-test is produced.

The assessment of the learning outcome based on the discussions with the students and the post-test shows that some learning objectives were reached in a measureable manner; however, it was just as satisfactory that most students became aware of the issues involved with the core elements of the course. They may not be able to fully “*describe the modal dynamics of three-bladed wind turbines*” in their sleep, but they know the concepts and where to find the information that they need. As mentioned in the introduction to Assignment 4A, the conceptual understanding of the modal dynamics of wind turbine are causing problems even for people working with wind turbine dynamics for many years, and I had the feeling after the course that the number of people within the wind energy industry able to discuss these issues had increased dramatically over just one week.

2.4 Analysis of course evaluation by students

The short course period made a midterm evaluation irrelevant because there would be little time for changing the course. Ph.D. students and paying participants from the industry gave (and were encouraged to give) spontaneous feedback when they fell that something can be done in another way, and this feedback let to a few changes along the way. An example is the feedback on the unclear and lacking prerequisites that led to the whiteboard presentation of the theory of eigenvalue analysis, which will become part of the next course.

On the last day of the course the students were asked to fill out the Course Experience Questionnaire. The calculated averaged and actual scores are shown in the following two diagrams (only 29 answers due to one absent student):



2.4.1 Good teaching

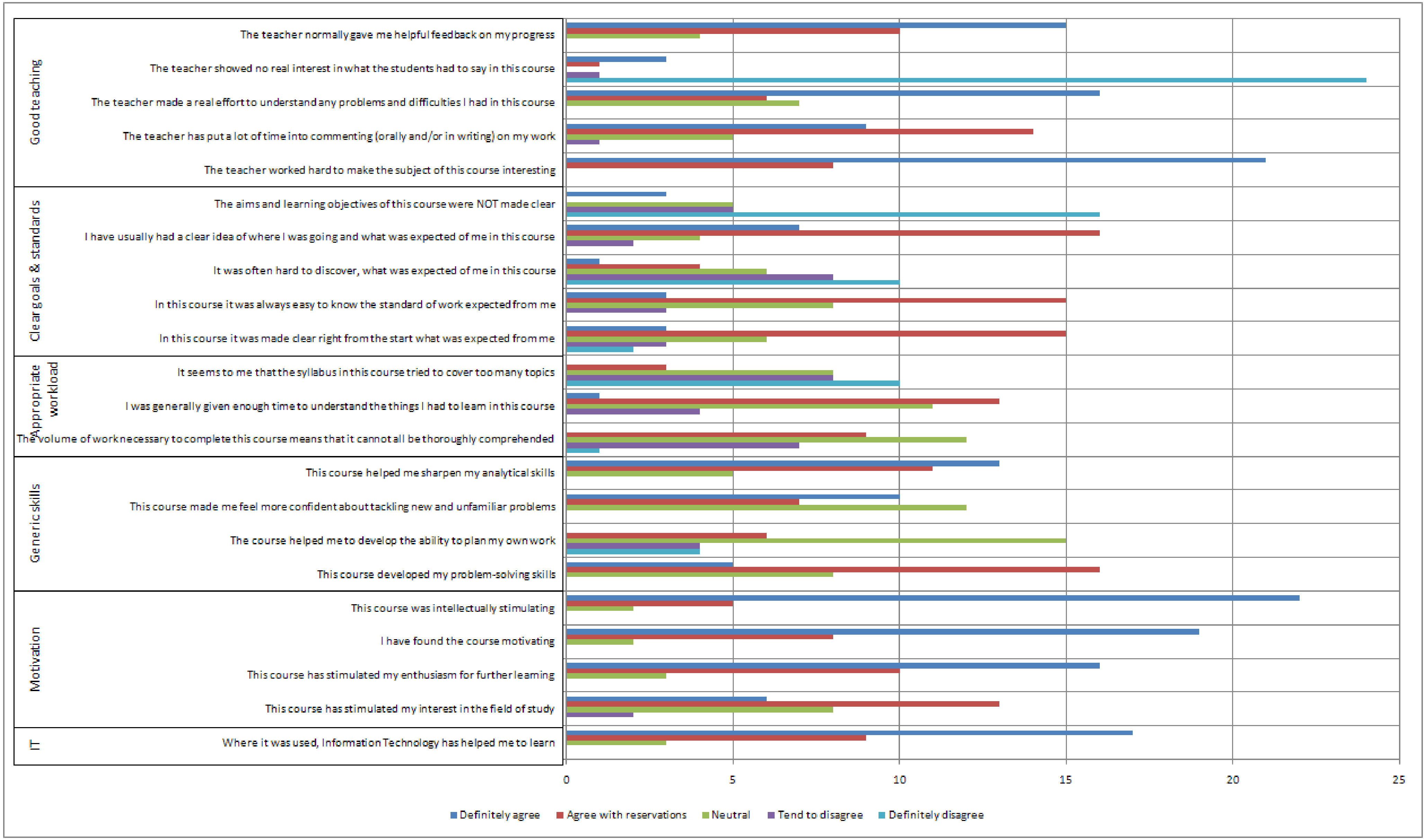
There is a relative high averaged score on the indicators for my ability to contribute to student learning. Looking at the actual answers to each indicator, there is an important restriction on this high score: Three students "definitely agree" that I showed no real interests in what they had to say. It can be assumed that they didn't overlook the word "no" in the question, because the scores on the two other questions dealing with my understanding of their problems and difficulties and my time used on commenting their work are also lower than the averaged score. I believe that the larger than planned number of students made it difficult to reach all students during the discussions in the class-room and during the exercise. In the four hours each afternoon of exercise, there was very few minutes where I or my colleague were not involved in discussions, but we may not have been good enough to make sure that all members of each group felt that their problems and difficulties were part of these discussions.

2.4.2 Clear goals and standards

There is a relative lower averaged score on the indicators for the clarity of the objectives and requirements to the students. The actual answers show that it may not have a problem with explaining the aims and learning objectives of the course (although three students definitely agree that these were not made clear). The problem seems more to be that the expectations of them were unclear to the students, and the written feedback, based on a DCAMM questionnaire discussed in the following, also showed that the objectives and standards for the exercise were unclear to them in the beginning. This confusion may also have been caused by the unclear prerequisites for the course and the change in ambitions for the exercise, as discussed earlier.

2.4.3 Appropriate workload

The lowest averaged score is on the indicators for the workload put on the students. One reason for this low score is probably caused by the workload of the exercise, e.g. in the programming of their own code as indicated by the written feedback described below.



Good teaching
The teacher normally gave me helpful feedback on my progress
The teacher showed no real interest in what the students had to say in this course
The teacher made a real effort to understand any problems and difficulties I had in this course
The teacher has put a lot of time into commenting (orally and/or in writing) on my work
The teacher worked hard to make the subject of this course interesting
Clear goals & standards
The aims and learning objectives of this course were NOT made clear
I have usually had a clear idea of where I was going and what was expected of me in this course
It was often hard to discover, what was expected of me in this course
In this course it was always easy to know the standard of work expected from me
In this course it was made clear right from the start what was expected from me
Appropriate workload
It seems to me that the syllabus in this course tried to cover too many topics
I was generally given enough time to understand the things I had to learn in this course
The volume of work necessary to complete this course means that it cannot all be thoroughly comprehended
Generic skills
This course helped me sharpen my analytical skills
This course made me feel more confident about tackling new and unfamiliar problems
The course helped me to develop the ability to plan my own work
This course developed my problem-solving skills
Motivation
This course was intellectually stimulating
I have found the course motivating
This course has stimulated my enthusiasm for further learning
This course has stimulated my interest in the field of study
IT
Where it was used, Information Technology has helped me to learn
0
5
10
15
20
25
Definitely agree
Agree with reservations
Neutral
Tend to disagree
Definitely disagree

Another reason is the high complexity of the taught subjects, and the steep learning curve needed for learning these subjects in such short and intense course. The level of the course can be better balanced, but it is expected (and hoped) that the students will fell a high workload due to the level in a Ph.D. course. The feeling of a high workload due to unnecessary programming efforts in the exercise is not intended because the course is not a programming course, and improvements to reduce this workload are discussed later.

2.4.4 Generic skills

There is also a relative low averaged score on the indicators for development of generic skills during the course. The actual answers show that the students fell that their analytical and problem-solving skills have been "sharpened", whereby I assume that they mean within the dynamics and aeroelasticity of wind turbines. The relative low averaged score may be caused by the many "neutral" answers to the other two questions in this group of indicators. These questions concerns the development of the students' generic skills on a broader level, which fits well in the evaluation of a normal university course, but which makes less sense in an intense Ph.D. course as the present, and I assume that the students therefore answered these questions neutrally.

2.4.5 Motivation

The relative highest averaged score is on the indicators for my ability to motivate the students and encourage further learning (on this subject). Looking at the actual answers shows that more than half the students answered "definitely agree" to the first three questions about motivation, whereas the last question about stimulation of the students' "interest in the field of study" scored a significantly lower average. This last question is unclear to me; what "field of study" is it referring to? If it refers to studies in general, a low score may just indicate that the participating Ph.D. students are already very interested in studying, and the industrial participants would not have a reason to become more interested in studying. If refers to this particular study subject of wind turbine dynamics and aeroelasticity then this lower averaged score would need attention. Anyway, it is unclear how the students understood this question.

2.4.6 IT

The relative high averaged score on the question regarding the use of IT in the learning process may just illustrate the fact that the exercise, and thereby the hands-on experience, was based on programming the methods and analyzing the results on a computer.

Overall, the evaluation based on the Course Experience Questionnaire points towards areas that need improvement. The DCAMM research school has also their own course evaluation form (see Appendix F) that the students filled out by after the UdTU questionnaire. The DCAMM evaluation form contains mainly written feedback, and the students were very helpful in writing positive feedback and suggestions for improvements[2]. A summary of these suggestions is directly transferred to the following section on future improvements.

[2] The filled out evaluation forms were send to DCAMM, unfortunately without making copies, except that I wrote a summary of the suggested improvements. The forms can be provided on request.

2.5 Future improvements

The improvements suggested by the students can be divided into five areas: the prerequisites, the teaching methods and material, the course content, the exercise, and the practical matters. These suggestions fitted well with my own thoughts about the future improvements of the course as presented here.

2.5.1 Prerequisites

The Course Experience Questionnaire showed that the prerequisites and requirements for the course were unclear, and several students also elaborated on this uncertainty in their written feedback. Some of these comments also stated that the level of the prerequisites was too high; however, this feeling may be caused by the uncertainty of the level due to the vaguely formulated prerequisites. The following improvements will be made for the next course:

- More detailed description of the prerequisites in the flyer and other material sent out to the students.
- New pre-test for improved check of the students' prerequisites to design the brush-up teaching of these prerequisites at course begin.

2.5.2 Teaching methods and material

The course notes were sent out a week before the course started, and some students commented that they could have been sent out earlier to provide more time for preparation. Many of the students suggested that these notes included a primer on the prerequisites concerning linear vibration and eigenvalue analysis. The students also asked for handouts of the slides, which only were provided for download after the course. They also asked for more complete teaching materials, for example a book instead of the three papers that were provided in the course notes.

Some students asked for more classical class-room teaching with derivation of theory on the whiteboard, for example the underlying mathematics of the methods used in the exercise. I feel that the content and structure of the class-room lectures and their part in course has not been well-prepared. The objectives and planning of the lectures need to be rethought, but I believe that the solution is not to make them more classic, I would rather integrated them more into the exercise from which the students seemed learn best. Future improvements of the teaching include:

- Alternative ways to integrate the exercise and class-room lectures more will be considered, for example by "*just-in-time learning*", starting by introducing the vibration problems of the test turbine Monday morning. This idea was considered earlier but discarded due to limited preparation time.
- The course notes will be sent out when the student has registered for the course.
- A primer note on linear vibration and eigenvalue analysis will be included in the course notes.
- Over time, the course notes will be written into a book(-let).

2.5.3 Course content

Some students suggested that the introduction of linear vibration and eigenvalue analysis become part of the course, i.e., part of its core elements, even if it would require a longer duration of the course. Other students also asked for a longer course in order to cover more general issues (e.g. unsteady aerodynamics and control strategies), include more subjects (e.g. closed-loop eigenvalue analysis), and/or go through the underlying mathematics of the methods used in the course. I am not sure that the content of the course should be extended, but a longer course may help students to better learning by having more time to brush-up the prerequisites, and more time to analyze and reflect on the results from the exercise. A longer course will also give me more time to be involved in the learning of all students, which was not the case in the present course according to a few low scores on some indicators for "Good teaching" in the Course Experience Questionnaire. The following improvements will be done in the future:

- A brush-up lecture and/or exercise at course begin on the prerequisite will be cooperated.
- Preparation of auxiliary lectures and/or exercises on more advanced subjects will be prepared. These lectures or exercises could easily be implemented in the concept of "just-in-time-learning".
- An extension of the course with one more week will be considered, but not before the effects of the other improvements have been evaluated after its next run.

2.5.4 Exercise

The objectives and concept of the exercise were unclear to some students in the beginning, which may be caused by the course outline, starting with class-room lectures and then using little time on introducing the exercise. If the vibration problems with the test turbine and the objectives of the exercise were introduced in more details as the very first thing Monday morning, these uncertainties may have been cleared, and such introduction will furthermore motivate the theory introduced in the lectures.

Several students asked for more time to work with the exercise, and to analyze and reflect on the results. Especially, the time spend on programming standard, but necessary (e.g. the matrix integrator) routines should be reduced by providing these routine in pre-written MatLab code, similar to the code already prepared.

Some students asked for a closer relation of the artificial test turbine to real life turbines. This change is however difficult because the simplicity of the test turbine is designed to reduce the modelling efforts (which are not part of the core elements) and thereby focus on the qualitative dynamics and aeroelastic stability of wind turbines within the available time. The suggestions lead to the following improvements:

- The vibration problems of the test turbine used in the exercise will be introduced in the very beginning of the course as motivation and clarification of the course objectives.
- Groups of only two persons will be formed to avoid a three-wheel and make sure that all students can participate in the work and discussions during the exercise. This improvement limits the number of students to 20 to ensure that there is time for student presentations by all groups.
- The exercise will be better balanced by providing the students with pre-written standard routines to make sure that all groups have time to complete the parts of the exercise needed for the student presentations.
- Topics for the student presentations will be better prepared based on the experiences from this course.

2.5.5 Practical matters

Some students asked for coordinated lunches to provide a framework for networking, which will be considered for the next run of the course. Some students wanted to break the four hour afternoon session in the databar into more parts, for example by spreading the exercise over the whole day with longer breaks of lectures. This change will be considered in connection with the integration of the class-room teaching and the exercise. Finally, the students complained that the databar was too hot, which will be discussed with DTU officials at the next course.

2.6 Effects of peer coaching

I met with my peer group in the week before the course to present the course plan and teaching methods. This presentation and discussion of my plans for the peer-group helped me see the course as a whole instead of working with it in parts. It became clear to me that I had spent most of my time preparing the exercise, and less on the lectures.

We agreed that two peer members attended the class-room lecture Wednesday morning to see what effect this priority had on the learning. The lecture dealt with aeroelastic modelling of wind turbines and derivation of the linear aeroelastic equations of motion for the test turbine used in the exercise. There was no buzz-groups planned for this lecture; however, questions were posed to the students whenever possible. The problem with the particular student interrupting me and the other students during this dialog was still not solved at the time, and I tried to reintroduce the rule of hand-raising. How to handle this problem became the main topic of the feedback. The peer-group suggested that I should discuss the problem with the student (as I already had), and agreed that hand-raising is a solution if his behaviour did not change.

2.7 Answers to my overall question

The carefully designed exercise seemed to be the answer to my overall question of how to teach theoretical difficult subjects to research oriented (the Ph.D. students) and problem oriented (the participants form industry) students. There are several conditions that may explain why it was a success: First, the exercise has a somewhat inductive approach by presenting a practical problem (the vibration problems of the test turbine), and letting the students first analyze the problem based their own programming of briefly introduced methods, and then present their own explanations to the problem and possible solutions to each other. Second, letting research and problem oriented students work together in mixed groups with each their own favourite places in Kolb's learning cycle makes them discuss across this cycle and thereby go all around it. Third, the subject of dynamics in a mechanical system like a wind turbine appeals to all mechanical engineers' intuition and experience, whereby a practical example as the present exercise almost makes the theory self-explanatory. Fourth, many students liked this subject and/or were working with related subjects, and they were therefore very motivated to learn.

These conditions may not be present the next time I give the course, e.g. I may not always have participants from the industry that make sure that there is focus on the problem, and not on the methods, in each group; or vice versa. I hope, however, that these effects of the exercise can be more predictable when lectures and exercises get more integrated, for example by using "just-in-time-learning".

2.8 Final reflections

This assignment is first of all a documentation of the experiences and suggestions for improvements that came out of giving this course. It will help me prepare for the next run of the course, or maybe help my colleagues that want to teach a similar course, or take over this one. The writing of the assignment has also helped me reflect more on my experiences, and many of the suggested improvements may not have been brought up without this reflection. Writing a similar, but shorter, document for each of my courses may be a good idea, especially for the analysis of the student feedback from the course evaluation questionnaires. It could help structure and remember the course improvements.

Finally, this assignment has helped me reflect on the collection of knowledge obtained from all the UdTU modules. Writing down this reflection may also help me remember what I have learned.

Appendix A – Course flyer (2 pages)

Course Description

Background: Modal frequencies, damping, and shapes of the vibration modes of operating wind turbines can be computed with and without the influence of the aerodynamic forces from the surrounding flow. The vibration modes of the unforced turbine (excluding the aerodynamic forces) constitute a dynamic fingerprint that defines its modal dynamics; these structural modes form the basis of the aeroelastic response of wind turbines due to the aerodynamic forces. The vibration modes of the turbine including the aerodynamic forces determine its aeroelastic stability properties through the damping of these aeroelastic modes. Stall-induced vibrations and classical flutter are the two main mechanisms that may lead to aeroelastic instabilities of three-bladed turbines with negative damping of an aeroelastic mode.

Learning objectives: At the end of the course the participant should be able to:

- Describe the methods of computing the modal frequencies, damping, and shapes of the structural and aeroelastic modes of operating wind turbines, and apply these methods for turbine analysis.
- Describe the modal dynamics of three-bladed wind turbines, and to identify these structural modes in the aeroelastic response of operating wind turbines.
- Describe the mechanisms of the aeroelastic instability called stall-induced vibrations, and explain which turbine parameters that determine the stability limits with respect to stall-induced vibrations.
- Describe the mechanisms of the aeroelastic instability called classical flutter, and explain which turbine parameters that determine the flutter limits.
- Work with state-of-the-art topics within the area of dynamics and aeroelastic stability of wind turbines.

The teaching will consist of a combination of dialog-based lectures and group work with illustrative example assignments, where the introduced methods are applied.

Course homepage

http://www.dcamm.dk.

Organizer

Senior Scientist Morten Hartvig Hansen, Wind Energy Department, Risø National Laboratory for Sustainable Energy, Technical University of Denmark.

This course is offered as part of the activities of the DCAMM International Graduate Research School, see www.dcamm.dk.

Participants

This course is aimed at graduate students interested in wind energy research and engineers in the wind turbine industry with purpose that they learn how to describe, analyze, and optimize the modal dynamics and aeroelastic stability of wind turbines based on the commercial three-bladed concept. The participants are presumed to have pre-knowledge of vibration analysis and linear dynamics of mechanical systems. No particular knowledge of wind turbines is needed; however, a general knowledge of structural dynamics and/or aerodynamics is presumed.

Working Load

Approximately 40 hours in total, including work during the June 23-27, 2008 course period at DTU (lectures, discussions, and assignments) as well as preparatory required reading before course start.

Tentative program outline

Monday, June 23, 2008: Methodology

08.00-09.00	Delegate Registration
09.00-09.30	Welcome and Introduction
09.30-12.00	Methods for vibration analysis of three-bladed turbines
12.00-13.00	Lunch
13.00-17.00	Group work with methods for vibration analysis of three-bladed turbines

Tuesday, June 24, 2008: Modal Dynamics

09.00-12.00	Modal dynamics of two- and three-bladed turbines
12.00-13.00	Lunch
13.00-17.00	Group work with modal dynamics of three-bladed turbines

Wednesday, June 25, 2008: Stall-induced Vibrations

09.00-12.00	Stall-induced vibrations of isolated blades and three-bladed turbines
12.00-13.00	Lunch
13.00-17.00	Group work with stall-induced vibrations

Thursday, June 26, 2008: Classical Flutter

09.00-12.00	Classical flutter of isolated blades and three-bladed turbines
12.00-13.00	Lunch
13.00-17.00	Group work with classical flutter
19:00-	Workshop Dinner

Friday, June 27, 2008: State of the art & visit to Risø

09.00-09.30	Arrival to Risø and breakfast together with the Aeroelastic Design group
09.30-12.00	Presentation of current research in aeroelasticity of wind turbines
12.00-13.00	Lunch
13.00-14.00	Visit to the old Test Station for Small Turbines
14.00-15.00	Course evaluation and discussion of future work

Study Material

Papers and lecture notes covering the course will be distributed by e-mail to the participants at the time of registration.

Language

All lectures will be given in English.

Evaluation and Diplomas

To pass the course, active participation and work with assignments are required. ETCS points: 2.5

Registration:

Ask for a registration form from the DCAMM-course secretariat, attn.: Kari Haugland, Department of Mathematics, Technical University of Denmark, Building 303S, DK-2800 Lyngby, Denmark. Tel.: (+45) 45253031, Fax: (+45) 45881399, E-mail: dcamm@mat.dtu.dk.

Registration fee:
There is no registration fee for students enrolled at universities and public research institutions. For researchers employed at universities and public research institutions the registration fee is 250 EURO. This covers hand-outs, coffee and social events. For all other participants the registration fee is 750 EURO.

Deadline:
Applicants should submit a request for registration to be at the hands of the course secretariat no later than **May 23rd, 2008.** Information on enrollment will be posted within a week after this date.

Housing:
There are a limited amount of rooms available on the premises of the Technical University of Denmark (DTU). These will be offered free of charge to students and otherwise at a cost of EURO 25 Euro per night. Accommodation in hostels/hotels can also be arranged by the participants themselves, see e.g. the Wonderful Copenhagen website at www.woco.dk.

Scholarships:
For Ph.D.-students enrolled at non-Danish universities and research institutions outside the EU, we can offer a limited number of scholarships in order to facilitate participation, covering lodging (see above) and extra living costs with a per diem amount of 25 EURO. Travel expenses will not be covered. Your CV and a short letter of recommendation from your Ph.D.-supervisor should be sent in together with the application form.

Internet Resources
For facts on the Technical University of Denmark and visitor's information: see http://www.dtu.dk. For information about teaching and research at the DCAMM departments: see http://www.dcamm.dk. For information about the Wind Energy Department at Risø-DTU, see http://www.risoe.dtu.dk/vea

About DCAMM
The **Danish Center for Applied Mathematics and Mechanics, DCAMM** is an informal framework for internationally oriented scientific collaboration between staff members at a number of departments at the Technical University of Denmark (DTU) and Aalborg University (AAU). The departments cooperating within DCAMM are:

- Dept. of Informatics & Mathematical Modelling, DTU
- Dept. of Mathematics, DTU
- Dept. of Mechanical Engineering, DTU
- Dept. of Civil Engineering, AAU
- Dept. of Mechanical Engineering, AAU

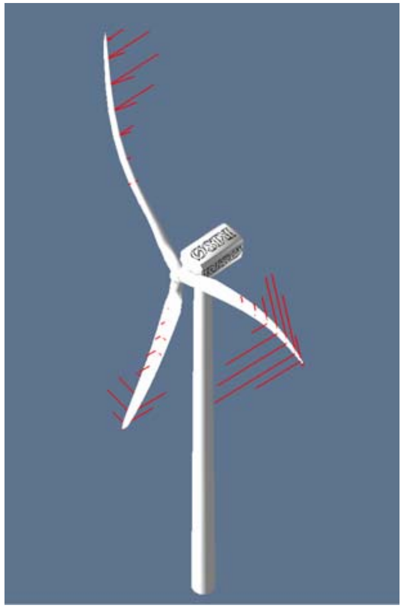
DCAMM is an informal construction. The day to day activities are coordinated by the secretary of the Center, while the formal governing body of DCAMM is the Scientific Council.

The **DCAMM International Graduate Research School** functions within the standard framework of the Ph.D.-education at the Technical University of Denmark (DTU) and at Aalborg University (AAU). Ph.D.-students associated to the School are full members of DCAMM through their departments and are enrolled in relevant Ph.D. programmes at DTU and AAU.

The School's role is to provide for an interdisciplinary framework for education of young researchers in an international research environment, and the activities are supported by Danish Agency for Research, Technology and Innovation.

DANISH CENTER FOR APPLIED MATHEMATICS AND MECHANICS

Ph.D.-course / Advanced school



Wind Turbine Dynamics and Aeroelasticity

Risø National Laboratory for Sustainable Energy
Technical University of Denmark,
Wind Energy Department,

Kgs. Lyngby, Denmark
June 23-27, 2008

Appendix B – Course notes (3 pages)

Wind Turbine Dynamics and Aeroelasticity

DCAMM Ph.D. course, June 23 – 27, 2008
Morten Hartvig Hansen and Bjarne Skovmose Kallesøe

This course deals with modal dynamics and aeroelastic stability of horizontal axis wind turbines. Modal frequencies, damping, and shapes of the vibration modes of operating wind turbines can be computed with and without the influence of the aerodynamic forces from the surrounding flow. The vibration modes of the unforced turbine (excluding the aerodynamic forces) constitute a dynamic fingerprint that defines its modal dynamics; these structural modes form the basis of the aeroelastic response of wind turbines due to the aerodynamic forces. The vibration modes of the turbine including the aerodynamic forces determine its aeroelastic stability properties through the damping of these aeroelastic modes. Stall-induced vibrations and classical flutter are the two main mechanisms that may lead to aeroelastic instabilities of three-bladed turbines with negative damping of an aeroelastic mode.

Learning Objectives

This course is aimed at graduate students interested in wind energy research and engineers in the wind turbine industry with purpose that they learn how to describe, analyze, and optimize the modal dynamics and aeroelastic stability of wind turbines based on the commercial three-bladed concept. At the end of the course the participant should therefore be able to:

- Describe the methods of computing the modal frequencies, damping, and shapes of the structural and aeroelastic modes of operating wind turbines, and apply these methods for turbine analysis.
- Describe the modal dynamics of three-bladed wind turbines, and to identify these structural modes in the aeroelastic response of operating wind turbines.
- Describe the mechanisms of the aeroelastic instability called stall-induced vibrations, and explain which turbine parameters that determine the stability limits with respect to stall-induced vibrations.
- Describe the mechanisms of the aeroelastic instability called classical flutter, and explain which turbine parameters that determine the flutter limits.
- Work with state-of-the-art topics within the area of dynamics and aeroelastic stability of wind turbines.

To reach these objectives you will work in groups of 2–3 persons with modal analysis and aeroelastic stability of a test turbine. In this exercise, you will program (in MatLab, or other mathematical/numerical tools) a numerical tool for computing the modal properties (natural frequencies, damping ratios, and mode shapes) and aeroelastic stability properties (aeroelastic frequencies, damping ratios, and mode shapes including aerodynamic forces) of the test turbine based on the theory and methods that you hear about in lectures.

Timetable

This timetable is tentative (we will put in breaks where needed).

	Monday	Tuesday	Wednesday	Thursday	Friday
8.00 – 9.00	Registration & breakfast (421/002)*	Breakfast (421/002)	Breakfast (421/002)	Breakfast (421/002)	Arrival at Risø **
9.00 – 10.00	Welcome and course introduction (421/002)	Discussion of yesterdays results: Modal dynamics of the blade (421/002)	Discussion of yesterdays results: Modal dynamics of the turbine (421/002)	Discussion of yesterdays results: Aeroelastic stability for the blade (421/002)	Breakfast together with the Aeroelastic Design Group (Risø)
10.00 – 12.00	Wind turbine terminology, introduction of the exercise with test turbine, and derivation of the linear structural equations of motion for test turbine (421/002)	Modal dynamics of wind turbines including methods for handling periodic terms in the equations of motion (421/002)	Aeroelastic modeling of wind turbines and derivation of the linear aeroelastic equations of motion for test turbine (421/002)	Mechanisms of aeroelastic instabilities relevant for wind turbines: Stall-induced vibrations and classical flutter (421/002)	Discussion of yesterdays results: Aeroelastic stability for the turbine, and perspective of the exercise in light of current research (Risø)
12.00 – 13.00	Lunch	Lunch	Lunch	Lunch	Lunch
13.00 – 17.00	Group work with Assignment A on modal dynamics of the blade (414/databar)	Group work with Assignment A on modal dynamics of the turbine (414/databar)	Group work with Assignment B on aeroelastic stability properties of the blade (414/databar)	Group work with Assignment B on aeroelastic stability properties of the turbine (414/databar)	13.00 – 14.00: Visit to the old Test Station for Small Turbines 14.00 – 15.00***: Course evaluation and discussion of future work
17.00 –				Pizzas and drinks	

* Locations of the different activities are written in parentheses as (building/room).
** Travel directions will be given during the course.
*** The course ends Friday at 15.00.

Literature

The elements of these learning objectives are covered by the three appended papers (Note that the original journal papers are available from www3.interscience.wiley.com). It is not required that you read all three papers before the course; however, you will probably be well-prepared and thereby gain more from the course if you read the review paper in Hansen (2007). The numerical tool that you are programming in the exercise is inspired by the structural model used in Hansen (2003), and you may learn something about aeroelastic stability analysis of wind turbines from Hansen (2004).

Bibliography

Hansen, M. H. (2003). Improved modal dynamics of wind turbines to avoid stall-induced vibrations, *Wind Energy* **6**: 179–195.

Hansen, M. H. (2004). Aeroelastic stability analysis of wind turbines using an eigenvalue approach, *Wind Energy* **7**: 133–143.

Hansen, M. H. (2007). Aeroelastic instability problems for wind turbines, *Wind Energy* **10**: 551–577.

Appendix C – Exercise notes (22 pages)

Group Exercise - A (worst) case study

DCAMM Ph.D. course, June 23 – 27, 2008
Wind Turbine Dynamics and Aeroelasticity
Morten Hartvig Hansen and Bjarne Skovmose Kallesøe

During this course you should imagine that you are working in a company that has started the design of a wind turbine where the preliminary simulations show large problems with vibrations. You must identify the source of these vibrations and suggest design improvements using the methods introduced in the course.

Due to time limitations, the levels of wind turbine modeling and design are restricted to the topics relevant to the learning objectives of the course. If you have a wind turbine background, or just use your common sense, you will find the design of the present turbine stupid and the assumptions of the modeling inadequate. The turbine and the modeling assumptions are designed such that you will have to analyze all different modal dynamics and aeroelastic instabilities seen on turbines, sometimes also on real ones. It is attempted to point out inadequate model and design issues when they arise, and part of your learning is also to discuss shortcomings when you see them; maybe you can conclude in the end of the exercise that the wind turbine only has problems in the simulations due to inadequate modeling and maybe not in real life.

The objective of this exercise is that you learn about modal dynamics and aeroelastic stability of wind turbines, and not that you learn how to derive the necessary structural and aeroelastic models. However, the best way to learn about dynamics of a system is to set up and work with the equations behind it. You may ask the instructors for hints and help to the modeling and the math whenever needed.

Practical issues

You will work in groups of 2–3 persons with the tasks given in this note. Monday and Tuesday are devoted to modal dynamics (Task A), and during Wednesday and Thursday you will work with the aeroelastic stability properties of the blades and turbine (Task B). Each group will have access to a PC with the necessary software (Maple, Mathematica, MatLab, C/Fortran compilers) to set up and analyze a linear structural and aeroelastic model of the turbine using the methods introduced in the course.

Each afternoon at 17.00 (except Friday), you must deliver your current results of the tasks. Results from all groups are then plotted against each other and presented the following day. There will be discrepancies and we will discuss the possible reasons for those. Please note that you will not be evaluated on correctness of your results, but on your active participation in the exercise and these discussions.

The Turbine

- The turbine designed for this exercise is a stall-regulated turbine with constant rotor speed of 1.5 rad/s (asynchronous induction generator) and fixed pitch angle of zero pitch.
- The blades (including the hub) are 50 m long, isotropic and prismatic with zero twist and constant 3 m chord length and thin airfoil thickness with symmetric profile.
- The nacelle has a mass of 205 ton, it is mounted on the tower without static tilt angle, and the distance from the tower top–drivetrain intersection to the rotor center is 5 m.
- The steel tower is 70 m high, isotropic and prismatic with outer diameter of 5 m and plate thickness of 40 mm.

Turbine parameters

Here is a list of measurable parameters for the blades, nacelle/drivetrain, and tower. Also shown are the estimated structural logarithmic decrements (note the large drivetrain damping that is added to model the effect of generator slip), and the air density used in the simulations.

Parameter	Description	Value	
R	Rotor radius / blade length	50	m
c	Blade chord length	3	m
a_{cg}	Distance from torsional point aft to center of gravity	1.2	m
EI_x	Flapwise bending stiffness	9.87975	GNm^2
EI_y	Edgewise bending stiffness	17.56404	GNm^2
GK	Torsional stiffness	0.1764804	GNm^2
m	Blade mass per unit-length	220	kg/m
J	Cross-sectional moment of inertia about center of gravity	275.75	kgm^2
L_s	Distance from tower top/drivetrain intersection to rotor center	5	m
G_s	Torsional stiffness of drivetrain	0.5	GNm/rad
M_n	Nacelle mass	205	ton
I_x	Tilt moment of inertia of nacelle	4500	tonm^2
I_y	Roll moment of inertia of nacelle	1200	tonm^2
I_z	Yaw moment of inertia of nacelle	4500	tonm^2
D	Outer diameter of tower	5.00	m
d	Inner diameter of tower	4.92	m
E_t	Young's modulus of tower steel	211	GPa
ν	Possion's ratio of tower steel	0.33	-
ρ_t	Density of tower steel	7850	kg/m^3
δ_f	Logarithmic decrement of first flapwise blade bending mode	0.01	-
δ_e	Logarithmic decrement of first edgewise blade bending mode	0.01	-
δ_t	Logarithmic decrement of first blade torsional mode	0.04	-
δ_{twr}	Logarithmic decrement of first tower bending modes	0.005	-
δ_{DT}	Logarithmic decrement of first drivetrain torsional mode	0.3	-
ρ	Air density	1.225	kg/m^3

Airfoil characteristics

The thin symmetric airfoil of the blades is assumed to have the following aerodynamic lift and drag coefficients

$$\begin{aligned} C_L(\alpha) &= 2\pi\alpha f(\alpha) + C_N(\alpha)\cos(\alpha)\,(1-f(\alpha)) \\ C_D(\alpha) &= C_D^{\mathrm{fric}} f(\alpha) + \tfrac{3}{4}C_N(\alpha)\sin(\alpha)\,(1-f(\alpha)) \end{aligned} \tag{1}$$

where C_D^{fric} is the drag in attached flow due to friction, and $C_N(\alpha)$ is the normal force coefficient for a flat plate in a fully separated flow [1]:

$$C_N(\alpha) = 2.25\frac{2\pi\sin(\alpha)}{4+\pi|\sin(\alpha)|} \tag{2}$$

Hence, the lift coefficient is a linear interpolation between attached flow value of $2\pi\alpha$ and fully separated flow value $C_N(\alpha)\cos(\alpha)$. The interpolation factor $f \in [0:1]$ is a function of the angle of attack defined as

$$f(\alpha) = \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{\alpha^{\mathrm{stall}} - |\alpha|}{\Delta\alpha^{\mathrm{stall}}}\right) \tag{3}$$

where α^{stall} and $\Delta\alpha^{\mathrm{stall}}$ define the stall characteristics. The factor f describes how separated the flow is: the flow is fully attached for $f = 1$, and fully separated for $f = 0$. Except of extremely thin airfoils, or airfoils undergoing rapid pitching, the flow separation initiates at the trailing edge and moves forward on the airfoil for higher angles of attack. The factor f can therefore be considered as a measure of the distance from the trailing edge to the separation point. Note that $f(\alpha^{\mathrm{stall}}) = 0.5$, thus α^{stall} defines an angle of attack for halfway developed stall, and $\Delta\alpha^{\mathrm{stall}}$ defines the width of the stall by the slope of the f function.

Figure 1 shows the lift and drag coefficients (1) with $C_D^{\mathrm{fric}} = 0.005$, α^{stall}=14 deg and $\Delta\alpha^{\mathrm{stall}}$=3 deg which are the values used in the following simulations.

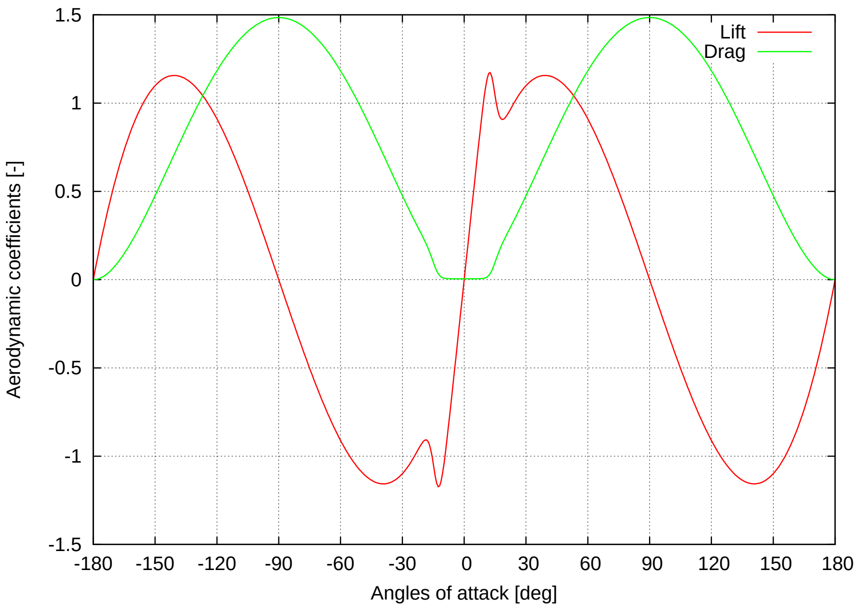


Figure 1: Lift and drag coefficients given by (1) with $C_D^{\mathrm{fric}} = 0.005$, α^{stall}=14 deg and $\Delta\alpha^{\mathrm{stall}}$=3 deg.

Simulations

Simulations with 5 % turbulence shows large vibrations of the test turbine at almost all wind speeds in range between 5–25 m/s. Figures 2–6 shows selected channels from these simulations at 8, 10, 12, 16, and 20 m/s.

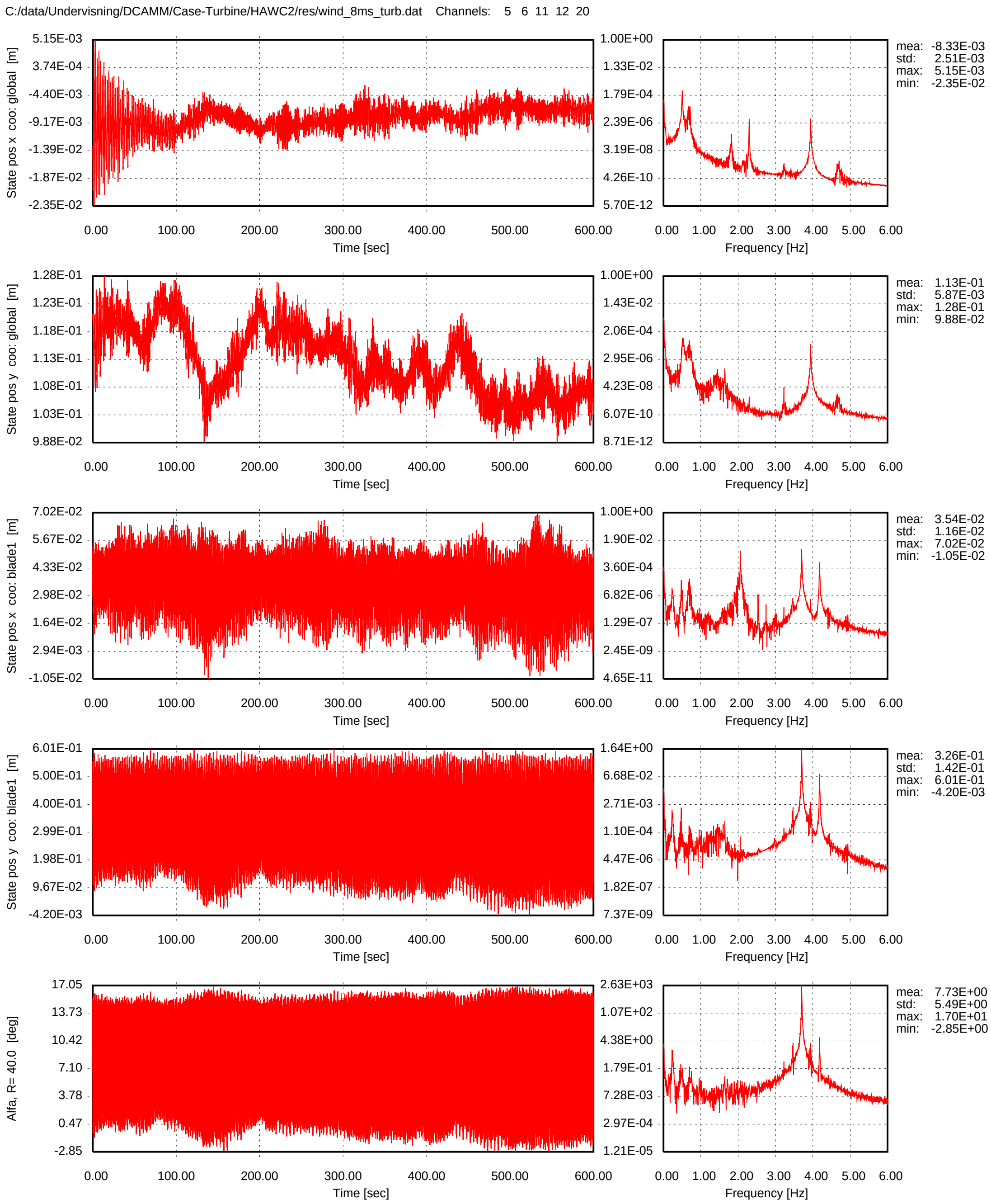


Figure 2: Simulations at 8 m/s. From top to bottom plot: Tower top lateral motion, Tower top longitudinal motion, Blade tip edgewise motion, Blade tip flapwise motion, and angle of attack at 39 m radius.

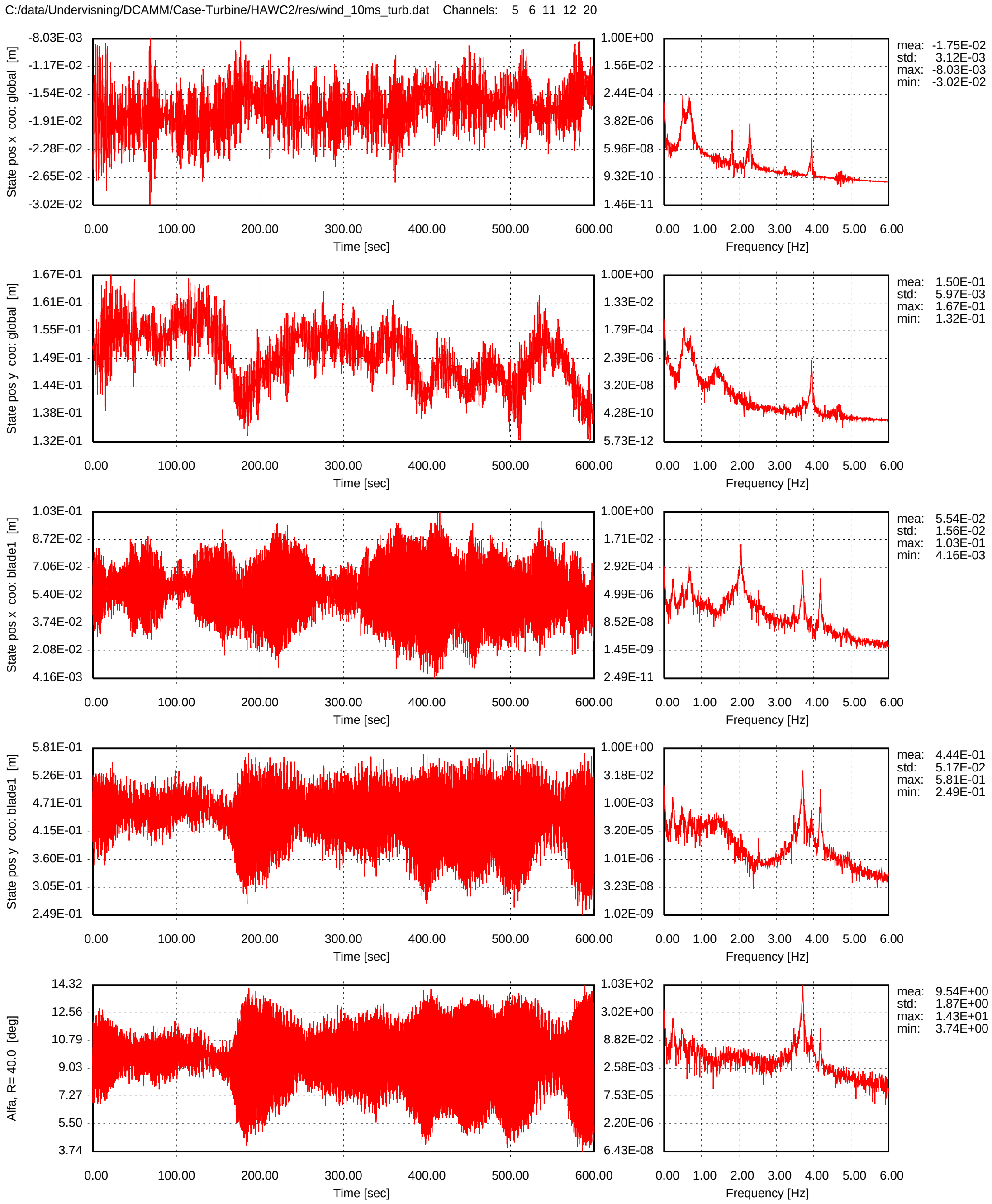


Figure 3: Simulations at 10 m/s. From top to bottom plot: Tower top lateral motion, Tower top longitudinal motion, Blade tip edgewise motion, Blade tip flapwise motion, and angle of attack at 39 m radius.

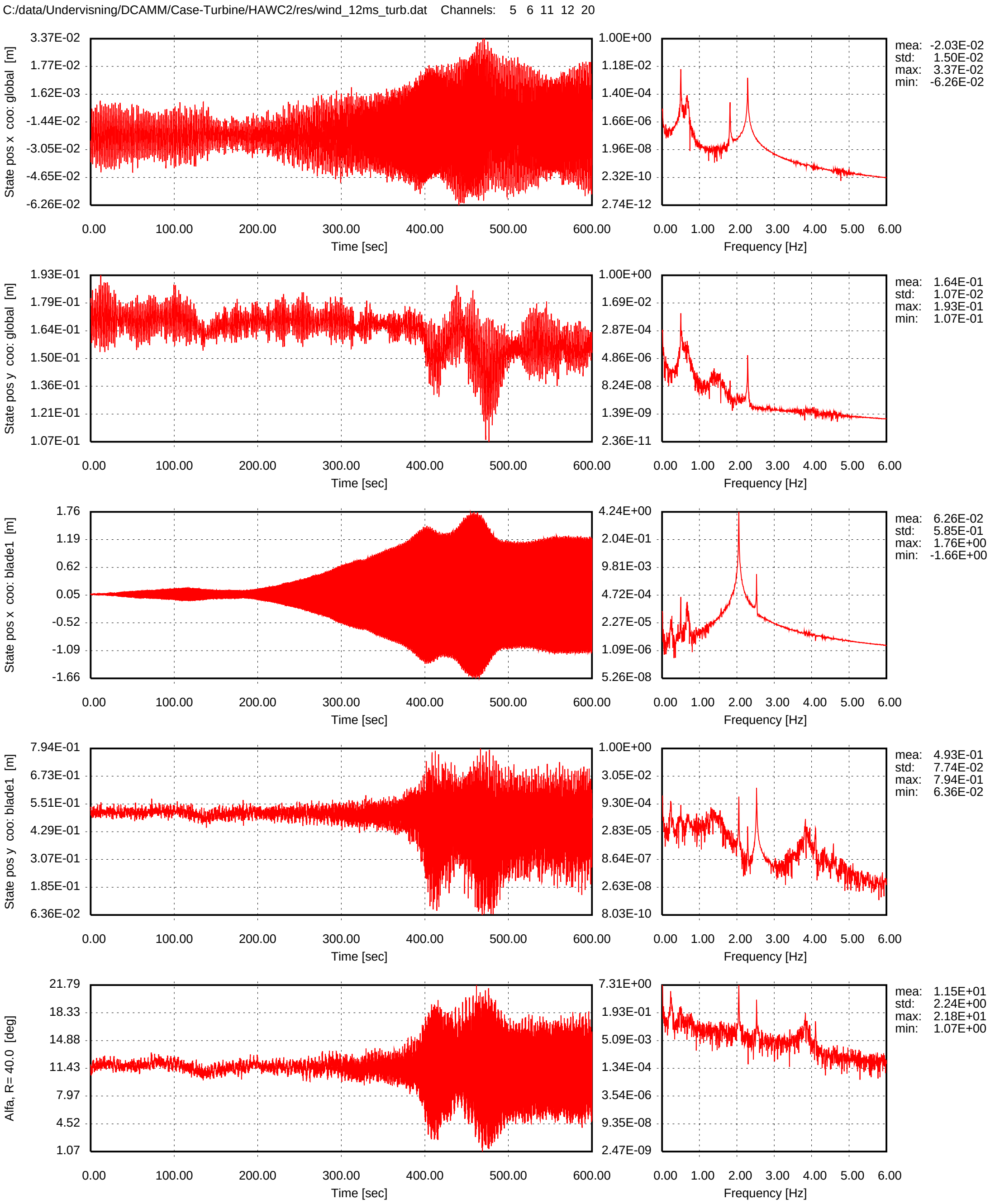


Figure 4: Simulations at 12 m/s. From top to bottom plot: Tower top lateral motion, Tower top longitudinal motion, Blade tip edgewise motion, Blade tip flapwise motion, and angle of attack at 39 m radius.

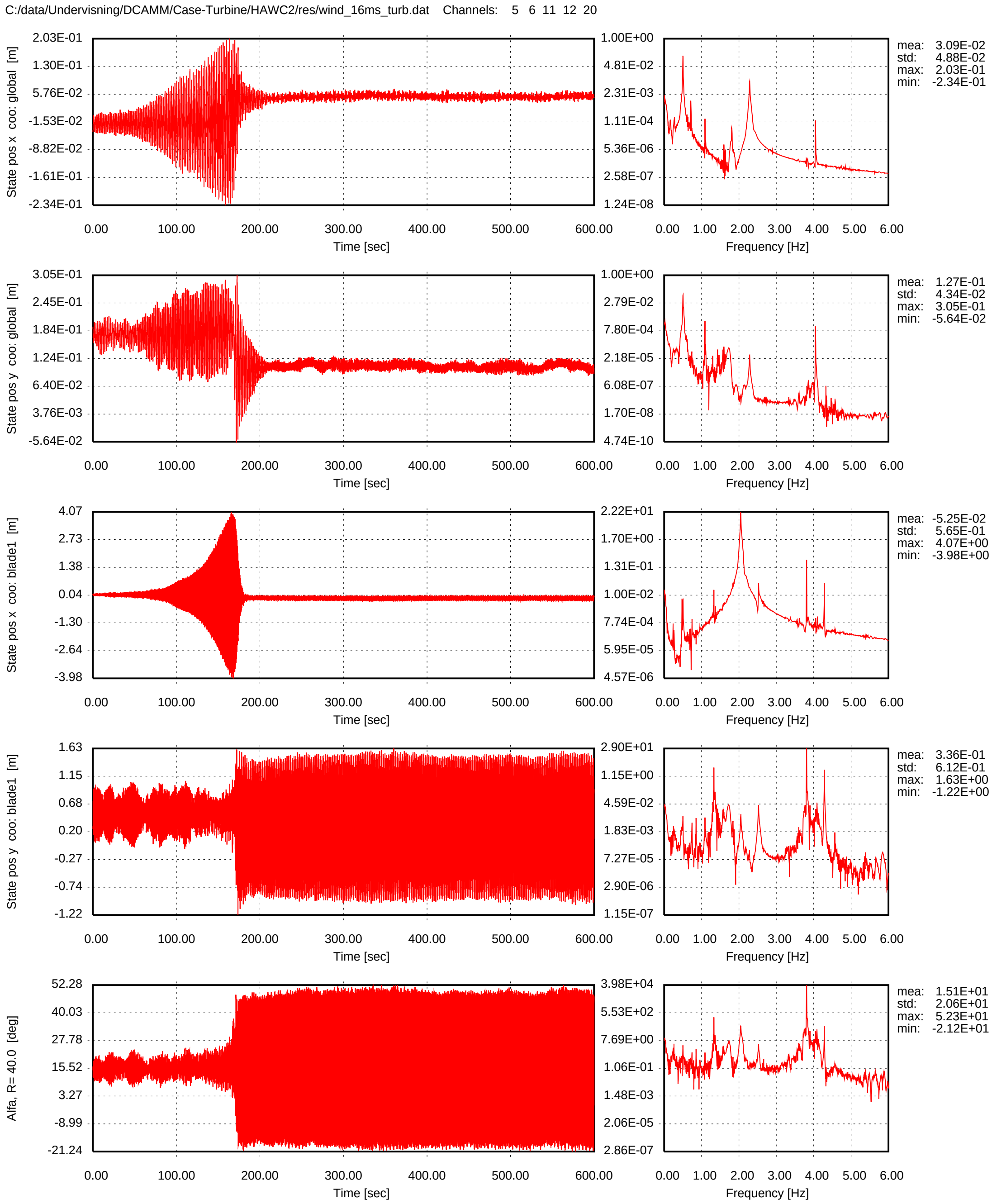


Figure 5: Simulations at 16 m/s. From top to bottom plot: Tower top lateral motion, Tower top longitudinal motion, Blade tip edgewise motion, Blade tip flapwise motion, and angle of attack at 39 m radius.

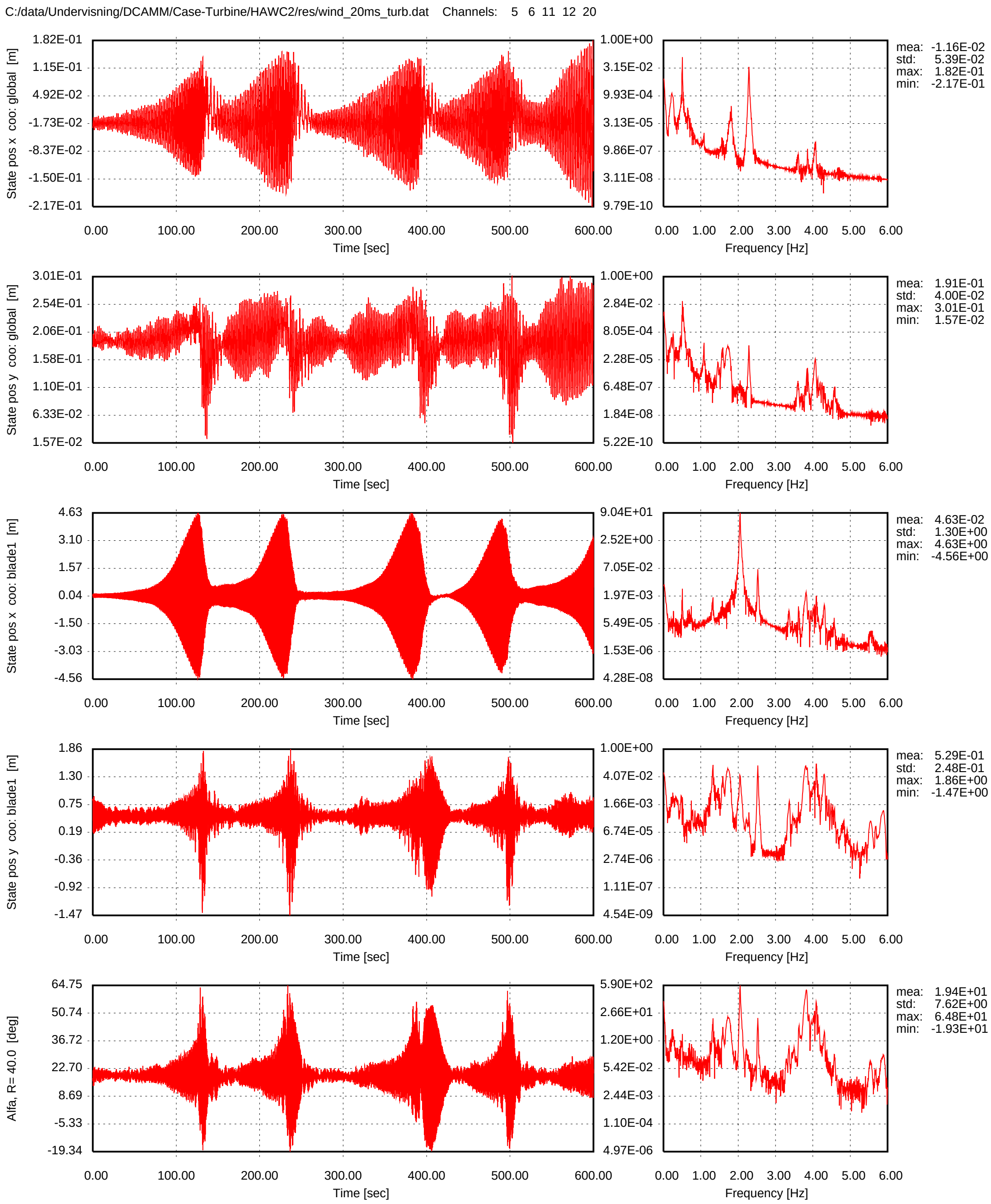


Figure 6: Simulations at 20 m/s. From top to bottom plot: Tower top lateral motion, Tower top longitudinal motion, Blade tip edgewise motion, Blade tip flapwise motion, and angle of attack at 39 m radius.

Task A – Modal Dynamics

The purpose of this task is to identify the modal content in the simulations of the test turbine. This identification can be done by comparing the peaks in the power spectra with computations of the natural frequencies of the blade and turbine modes.

Task A is divided into two subtasks: isolated blade analysis, and full turbine analysis, both based on the structural model provided at the end of this note. For the full turbine analysis you will need the model parameters listed in Table 1.

Lists of actions in the analyzes are now given, where • denotes mandatory investigations with deliverables, and ◦ denotes suggestions for further investigations.

Isolated blade analysis – Deadline Monday 17.00

- Compute the natural frequencies of the blade modes as function of rotor speed up to the rated rotor speed with steps of 0.05 rad/s. Provide your results in a file with rotor speed in the first column and ascending natural frequencies in the subsequent columns. Please use the sheet named "Blade frequencies" in Excel file "Task A-1 results group-color.xls", where "color" is replaced by your group color name.
- Compute the mode shapes of the blade modes for two rotor speeds: zero and the rated speed. Provide your results in two files, one for each rotor speed, with radius in the first column (with radial steps of 1 m) and pairs of amplitude and phase for the flapwise, edgewise, and torsional components of the mode shapes (2×3 columns per mode) with ascending natural frequencies in the subsequent columns. Please use sheets named "Blade mode shapes (standstill)" and "Blade mode shapes (rated speed)" in Excel file "Task A-1 results group-color.xls", where "color" is replaced by your group color name.
- Discuss your results: How do the natural frequencies compare to the simulations? How do some mode shapes couple, and why? How is the dependency of the modal properties on the rotor speed?

◦ Add structural damping to the blades, e.g. using a spectral damping model [2].

Parameter	Description	Value	
K_x	Lateral stiffness of nacelle support	14.15	MN/m
K_y	Longitudinal stiffness of nacelle support	14.15	MN/m
G_x	Tilt stiffness of nacelle support	23.11	GNm/rad
G_y	Roll stiffness of nacelle support	23.11	GNm/rad
G_z	Yaw stiffness of nacelle support	4.344	GNm/rad
g_{xy}	Coupling stiffness of nacelle support	-0.4953	GN/rad
M	Equivalent nacelle and tower mass	290.6	ton

Table 1: Additional model parameters computed by considering the tower as a cantilever beam and then using the provided formulas for common cases of an end force and moment on a cantilever beam to compute the stiffnesses. The additional mass from the tower to the nacelle motion is approximated as $\phi^{-2}(H) \int_0^H m_t \phi^2(x) dx$, where H is the tower height, m_t is the tower mass per unit-length, and $\phi(x)$ is the first bending mode shape function for a cantilever beam.

- ○ Make animations of the blade mode shapes.
- ○ Add additional DOFs: the second flapwise mode, or the pitch bearing DOF.

Full turbine analysis – Deadline Tuesday 17.00

- Compute the natural frequencies of the turbine modes as function of rotor speed up to the rated rotor speed with steps of 0.05 rad/s. Provide your results for the first 11 modes in a file with rotor speed in the first column and ascending natural frequencies in the subsequent columns. Please use the sheet named "Turbine frequencies" in Excel file "Task A-2 results group-color.xls", where "color" is replaced by your group color name.
- Compute the mode shapes of the turbine modes as function of rotor speed up to the rated rotor speed with steps of 0.05 rad/s. Provide your results for the first 11 modes in terms of nacelle motion and blade tips motion using the concept of symmetric and whirling rotor components, see Equation (18) in [3]. Your result file must have rotor speed in the first column, and then pairs of amplitude and phase for the following modal components: lateral and longitudinal translations (u_{tx} and u_{ty}), and the tilt, yaw, and roll motion (θ_{tx}, θ_{tz}, and θ_{ty}) of the nacelle (2×5 columns per mode), torsion (θ_{sy}) of the drivetrain (2 columns per mode), and the symmetric, forward and backward whirling components of the edgewise, flapwise, and torsional blade tip deflection (2×9 columns per mode). These 30 columns per mode must be arranged below each other with ascending natural frequencies. Please use the sheet named "Turbine mode shapes" in Excel file "Task A-2 results group-color.xls", where "color" is replaced by your group color name.
- Discuss your results: How do the natural frequencies compare to the simulations? How do some mode shapes couple, and why? How is the dependency of the modal properties on the rotor speed?
- ○ Add structural damping to the nacelle/tower top DOFs, e.g. using a spectral damping model [2].
- ○ Make snapshot plots, or animations of the turbine mode shapes.
- ○ Make a sorting algorithm that sorts the modes after their mode shapes.
- ○ Add additional DOFs: the second flapwise mode, or the pitch bearing DOF.

Task B – Aeroelastic Stability

The purpose of this task is first to identify the aeroelastic modes with negative damping (eigenvalues with positive real parts) that have induced the vibrations in the simulations of the test turbine. Second, the purpose is to investigate the mechanism of the aeroelastic instability/ies for thereby to suggest design changes, or just improvements of the aeroelastic model.

Task B is also divided into two subtasks: isolated blade analysis, and full turbine analysis, both based on the aeroelastic model provided at the end of this note.

Isolated blade analysis – Deadline Wednesday 17.00

- Compute the frequencies and damping ratios of the aeroelastic blade modes as function of wind speed in the range 5–25 m/s with steps of 1 m/s. Provide your results in a file with wind speed in the first column, and frequency and damping ratio (2 per mode) in the subsequent columns sorted after ascending frequencies. Please use the sheet named "Blade frequencies" in Excel file "Task A-1 results group-color.xls", where "color" is replaced by your group color name.
- Compute the mode shapes of the aeroelastic blade modes for 8 m/s and 16 m/s. Provide your results in two files, one for each wind speed, with radius in the first column (with radial steps of 1 m) and pairs of amplitude and phase for the flapwise, edgewise, and torsional components of the mode shapes (2×3 columns per mode) with ascending frequencies in the subsequent columns. Please use sheets named "Blade mode shapes (8 m per s)" and "Blade mode shapes (16 m per s)" in Excel file "Task A-1 results group-color.xls", where "color" is replaced by your group color name.
- Discuss your results: How do the aeroelastic frequencies compare to the structural natural frequencies of the blade? Are there changes in mode shape couplings, and why? Are there any aeroelastic instabilities, and if so, what are their mechanisms? Can possible instabilities explain the large vibrations in the simulations?

◦ Investigate the effects of changing the airfoil characteristics.

◦ Investigate the effects of moving the center of gravity fore and aft on the airfoil chord.

◦ Make snapshot plots, or animations of the aerodynamic forces during motion in an aeroelastic blade mode.

◦ Compute the radial distributions of the work done by the aerodynamic forces on the blade motion.

◦ Add the effects of dynamic stall on the aerodynamic forces to the aeroelastic model.

◦ Add the effects of shed vorticity (Theodorsen effects) on the aerodynamic forces.

Full turbine analysis – Deadline Thursday 17.00

- Compute the frequencies and damping ratios of the aeroelastic turbine modes as function of wind speed in the range 5–25 m/s with steps of 1 m/s. Provide your results for the first 11 modes in a file with wind speed in the first column, and frequency and damping ratio (2 per mode) in the subsequent columns sorted after ascending frequencies. Please use the sheet named "Turbine frequencies" in Excel file "Task B-2 results group-color.xls", where "color" is replaced by your group color name.

- Compute the mode shapes of the aeroelastic turbine modes for each wind speed. Provide your results for the first 11 modes in terms of nacelle motion and blade tip motion using the concept of symmetric and whirling rotor components, similarly to the results for the structural turbine modes. Please use the sheet named "Turbine mode shapes" in Excel file "Task B-2 results group-color.xls", where "color" is replaced by your group color name.

- Discuss your results: How do the aeroelastic frequencies compare to the structural natural frequencies of the turbine? Are there changes in mode shape couplings, and why? Do isolated blade analysis and full turbine analysis agree on aeroelastic stability? Has any aeroelastic instability changed, and if so, why?

- ◦ Investigate the effects of changing the airfoil characteristics.

- ◦ Investigate the effects of moving the center of gravity fore and aft on the airfoil chord.

- ◦ Investigate the effects of changing the edgewise, or torsional blade stiffness.

- ◦ Investigate the effects of decreasing the tilt and yaw stiffnesses (Can the turbine go into whirl flutter?).

- ◦ Make snapshot plots, or animations of the aerodynamic forces during motion in an aeroelastic turbine mode.

- ◦ Compute the symmetric and whirling components of the radial distributions of the work done by the aerodynamic forces on the blade motion.

Bibliography

[1] S. F. Hoerner and H. V. Borst. *Fluid-Dynamic Lift.* Hoerner Fluid Dynamics, New York, 1975.

[2] R. W. Clough and J. Penzien. *Dynamics of Structures.* McGraw-Hill, 1975.

[3] M. H. Hansen. Improved modal dynamics of wind turbines to avoid stall-induced vibrations. *Wind Energy*, 6:179–195, 2003.

[4] J. J. Thomsen. *Vibrations and Stability.* Springer Verlag, 2003.

[5] L. Meirovitch. *Methods of Analytical Dynamics.* McGraw-Hill, New York, 1970.

[6] J. Katz and A. Plotkin. *Low speed aerodynamics - From wing theory to panel methods.* McGraw-Hill, Inc., New York, 1991.

Linear aeroelastic equations of motion

The simplest possible structural model that captures the lower order modal dynamics of the test turbine has 15 degrees of freedom (DOFs), see Figure 7. Five DOFs describe the motion of the nacelle: lateral (u_{tx}) and longitudinal (u_{ty}) translations, and tilt (θ_{tx}), roll (θ_{ty}), and yaw (θ_{tz}) rotations. One rotational DOF describes the elastic torsion of the drivetrain (DT) at the rotor center (θ_{sy}) relative to the generator which is fixed to rotate at constant speed Ω (the whole drivetrain is rotating at this slow speed, the gearbox is neglected because the generator speed is constant and does not contribute with rotational inertia forces). The flapwise ($u_{y,k}$) and edgewise ($u_{x,k}$) translations and torsion (θ_k) of blade k are modal expanded:

$$u_{y,k}(z,t) = \phi_1(z)q_{1,k}(t) \qquad u_{x,k}(z,t) = \phi_2(z)q_{2,k}(t) \qquad \theta_k(z,t) = \phi_3(z)q_{3,k}(t) \tag{4}$$

where $\phi_1(z)$, $\phi_2(z)$ and $\phi_3(z)$ are the first flapwise bending, edgewise bending, and torsional mode shapes, respectively, for a prismatic beam [4]. The generalized coordinates $q_{1,k}(t)$, $q_{2,k}(t)$, and $q_{3,k}(t)$ for these modal deflections on each blade ($k = 1, 2, 3$) are the remaining nine DOFs. Note that more modes can be included in these modal expansions, e.g. the second flapwise bending mode would be an obvious extension.

The position vector in the ground fixed frame of reference (with origin at the tower top/drivetrain intersection) to the center of gravity in radius z of the blade k is

$$\mathbf{r}_{cg,k} = \begin{Bmatrix} u_{tx}(t) \\ u_{ty}(t) \\ 0 \end{Bmatrix} + \mathbf{T}_{nac}(t)\left(\begin{Bmatrix} 0 \\ -L_s \\ 0 \end{Bmatrix} + \mathbf{T}_{DT}(t)\mathbf{T}_{\psi_k}(t)\left(\begin{Bmatrix} u_{x,k}(z,t) \\ u_{y,k}(z,t) \\ z \end{Bmatrix} + \mathbf{T}_{\theta_k}(t)\begin{Bmatrix} -a_{cg} \\ 0 \\ 0 \end{Bmatrix}\right)\right) \tag{5}$$

where L_s is the distance from tower top/drivetrain intersection to rotor center, and a_{cg} is the chordwise distance from the torsional point to the center of gravity (see Figure 8). The transformation matrices are

$$\mathbf{T}_{nac} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{tx} & -\sin\theta_{tx} \\ 0 & \sin\theta_{tx} & \cos\theta_{tx} \end{bmatrix}\begin{bmatrix} \cos\theta_{tz} & -\sin\theta_{tz} & 0 \\ \sin\theta_{tz} & \cos\theta_{tz} & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\theta_{ty} & 0 & \sin\theta_{ty} \\ 0 & 1 & 0 \\ -\sin\theta_{ty} & 0 & \cos\theta_{ty} \end{bmatrix},$$

$$\mathbf{T}_{DT} = \begin{bmatrix} \cos\theta_{sy} & 0 & \sin\theta_{sy} \\ 0 & 1 & 0 \\ -\sin\theta_{sy} & 0 & \cos\theta_{sy} \end{bmatrix}, \mathbf{T}_{\psi_k} = \begin{bmatrix} \cos\psi_k & 0 & \sin\psi_k \\ 0 & 1 & 0 \\ -\sin\psi_k & 0 & \cos\psi_k \end{bmatrix}, \mathbf{T}_{\theta_k} = \begin{bmatrix} \cos\theta_k & -\sin\theta_k & 0 \\ \sin\theta_k & \cos\theta_k & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{6}$$

where $\psi_k = \Omega t + \frac{2\pi}{3}(k-1)$ is the mean azimuth angle to blade k. Matrix $\mathbf{T}_{nac}$ handles the tilt, yaw, and roll of the nacelle, matrices $\mathbf{T}_{DT}$ and $\mathbf{T}_{\psi_k}$ handle the drivetrain torsion and rotor rotation of blade k, and

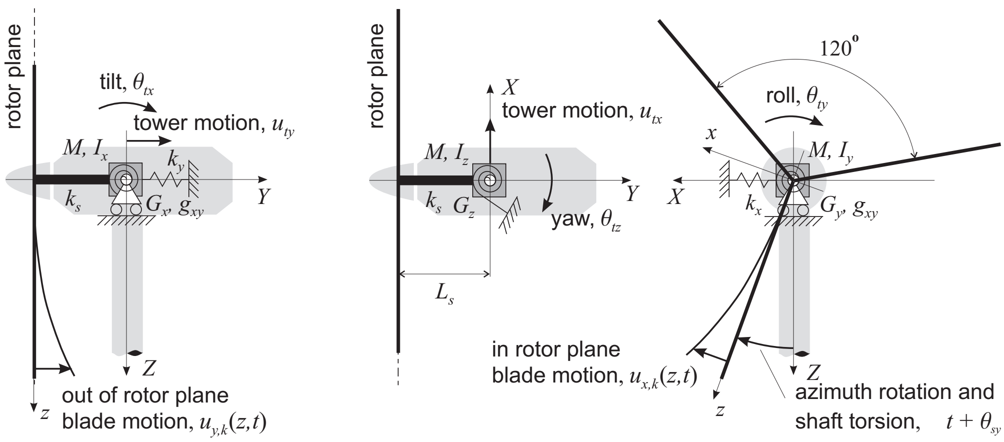


Figure 7: Schematics of structural turbine model.

matrix $\mathbf{T}_{\theta_k}$ handles the torsion of the blade chord. Note that the use of Euler angles is valid due to the following linearization about the zero steady state $\theta_{tx} = \theta_{ty} = \theta_{tz} = \theta_{sy} = \theta_k = 0$.

The equations of motion for a discrete dynamic system of size N can be derived from Lagrange's equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i} + \frac{\partial D}{\partial \dot{x}_i} = Q_i \quad \text{for} \;\; i = 1, \ldots, N \tag{7}$$

where $L = T - V$ is the Lagrangian given by the kinetic energy T and the potential energy V of the conservative forces, $\dot{x} = dx_i/dt$ is the time derivative of the generalized coordinate x_i, D is Rayleigh's dissipation function $D = \dot{\mathbf{x}}^T \mathbf{C}\,\dot{\mathbf{x}}$ given by the symmetric damping matrix $\mathbf{C}$ used to model the internal energy dissipation in the system as viscous damping (see e.g. [4, 5]), and Q_i is the generalized force for the generalized coordinate x_i which is derivable from the non-conservative forces. The quadratic property of Rayleigh's dissipation function with a constant $\mathbf{C}$ matrix ensures linear damping forces in the equations of motion through the term $\partial D/\partial \dot{x}_i$ in Lagrange's equations.

The total kinetic energy of the test turbine can be written as:

$$T = \tfrac{1}{2}M\left(\dot{u}_{tx}^2 + \dot{u}_{ty}^2\right) + \tfrac{1}{2}I_x\dot{\theta}_{tx}^2 + \tfrac{1}{2}I_z\dot{\theta}_{tz}^2 + \tfrac{1}{2}I_y\theta_{ty}^2 + \tfrac{1}{2}\sum_{k=1}^{3}\left(\int_0^R m\,\dot{\mathbf{r}}_{cg,k}\cdot\dot{\mathbf{r}}_{cg,k} + J\,\dot{\theta}_k^2\,dz\right) \tag{8}$$

and the potential energy can be written as:

$$\begin{aligned} V \;&=\; \tfrac{1}{2}K_x u_{tx}^2 + \tfrac{1}{2}K_y u_{ty}^2 + \tfrac{1}{2}G_x\theta_{tx}^2 + \tfrac{1}{2}G_y\theta_{ty}^2 + \tfrac{1}{2}G_z\theta_{tz}^2 - g_{xy}\theta_{tx}u_{ty} + g_{xy}\theta_{ty}u_{tx} + \tfrac{1}{2}G_s\theta_{sy}^2 \\ &+\; \tfrac{1}{2}\sum_{k=1}^{3}\left(\int_0^R\left(EI_y(u_{x,k}'')^2 + EI_x(u_{y,k}'')^2 + GK(\theta_k')^2 + \int_z^R m\Omega^2\zeta\,d\zeta\,\left((u_{x,k}')^2 + (u_{y,k}')^2\right)\right)dz\right) \end{aligned} \tag{9}$$

where $()' = d/dz$, and the potential energy due to the centrifugal normal force $\int_z^R m\Omega^2\zeta\,d\zeta$ is included. Insertion of $L = T - V$ into (7) and linearization about zero steady state ($\mathbf{x} = \dot{\mathbf{x}} = \mathbf{0}$) yields the linear equations of motion:

$$\begin{aligned} &\begin{bmatrix} \mathbf{M}_b & \mathbf{0} & \mathbf{0} & \mathbf{M}_{bt}(\psi_1) \\ \mathbf{0} & \mathbf{M}_b & \mathbf{0} & \mathbf{M}_{bt}(\psi_2) \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_b & \mathbf{M}_{bt}(\psi_3) \\ \mathbf{M}_{tb}(\psi_1) & \mathbf{M}_{tb}(\psi_2) & \mathbf{M}_{bt}(\psi_3) & \mathbf{M}_t \end{bmatrix}\ddot{\mathbf{x}} + \begin{bmatrix} \mathbf{C}_b & \mathbf{0} & \mathbf{0} & \mathbf{G}_{bt}(\psi_1) \\ \mathbf{0} & \mathbf{C}_b & \mathbf{0} & \mathbf{G}_{bt}(\psi_2) \\ \mathbf{0} & \mathbf{0} & \mathbf{C}_b & \mathbf{G}_{bt}(\psi_3) \\ \mathbf{G}_{tb}(\psi_1) & \mathbf{G}_{tb}(\psi_2) & \mathbf{G}_{tb}(\psi_3) & \mathbf{G}_t + \mathbf{C}_t \end{bmatrix}\dot{\mathbf{x}} \\ &\qquad + \begin{bmatrix} \mathbf{K}_b & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_b & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_b & \mathbf{0} \\ \mathbf{K}_{tb}(\psi_1) & \mathbf{K}_{tb}(\psi_2) & \mathbf{K}_{tb}(\psi_3) & \mathbf{K}_t \end{bmatrix}\mathbf{x} = \mathbf{F}_0 + \mathbf{Q} \end{aligned} \tag{10}$$

where the vector of generalized coordinates/DOFs $\mathbf{x}$ is

$$\mathbf{x} = \{\underbrace{q_{1,1}\,q_{2,1}\,q_{3,1}}_{\text{Blade 1}}\;\underbrace{q_{1,2}\,q_{2,2}\,q_{3,2}}_{\text{Blade 2}}\;\underbrace{q_{1,3}\,q_{2,3}\,q_{3,3}}_{\text{Blade 3}}\;\underbrace{u_{tx}\,u_{ty}\,\theta_{tx}\,\theta_{tz}\,\theta_{ty}\,\theta_{sy}}_{\text{Nacelle + DT}}\}^T, \tag{11}$$

the steady state conservative force vector $\mathbf{F}_0 = -m\Omega^2 a_{cg}\int_0^R \phi_2 dz\{0\,1\,0\,0\,1\,0\,0\,1\,0\,0\,0\,0\,0\,0\,0\}^T$, and the generalized force vector is $\mathbf{Q} = \{Q_1 \ldots Q_i \ldots Q_{15}\}^T$. All system matrices in (10) are listed at end of this note. The viscous damping matrices for each blade $\mathbf{C}_b$ and the nacelle/tower motion $\mathbf{C}_t$ may be set up using the spectral damping model [2]. It is noticed that the coupling matrices with subscripts $_{bt}$ and $_{tb}$ depend on the mean azimuth angle $\psi_k = \Omega t + \frac{2\pi}{3}(k-1)$ to the particular blade k.

The generalized forces Q_i in (7) due to the non-conservative aerodynamic forces are defined by

$$Q_i = \sum_{k=1}^{3} \left(\int_0^R \frac{\partial \mathbf{r}_{ac,k}(z,\mathbf{x})}{\partial x_i} \cdot \mathbf{F}_k(z,\mathbf{x},\dot{\mathbf{x}}) dz \right) \quad \text{for } \; i = 1, \ldots, 15 \tag{12}$$

where $\mathbf{r}_{ac,k}$ is the position vector of the aerodynamic center (ac) in the ground fixed frame along blade k:

$$\mathbf{r}_{ac,k}(z,\mathbf{x}) = \left\{ \begin{array}{c} u_{tx} \\ u_{ty} \\ 0 \end{array} \right\} + \mathbf{T}_{nac} \left(\left\{ \begin{array}{c} 0 \\ L_s \\ 0 \end{array} \right\} + \mathbf{T}_{DT} \mathbf{T}_{\psi_k} \left\{ \begin{array}{c} u_{x,k} \\ u_{y,k} \\ z \end{array} \right\} \right) \tag{13}$$

and $\mathbf{F}_k$ is the aerodynamic forces along blade k in the ground fixed frame given as

$$\mathbf{F}_k(z,\mathbf{x},\dot{\mathbf{x}}) = \mathbf{T}_{nac} \mathbf{T}_{DT} \mathbf{T}_{\psi_k} \left\{ \begin{array}{c} f_{x,k}(z,\mathbf{x},\dot{\mathbf{x}}) \\ f_{y,k}(z,\mathbf{x},\dot{\mathbf{x}}) \\ 0 \end{array} \right\} \tag{14}$$

where $f_{x,k}$ and $f_{y,k}$ are the aerodynamic forces along blade k in its rotating frame of reference. Notice that the aerodynamic moment about the aerodynamic center is assumed to be zero which is valid for attached flow, but not in stall. However, the aerodynamic moment is not important for the qualitative understanding of the aeroelasticity of the test turbine and is therefore neglected.

The aerodynamic forces along blade k in its rotating frame of reference are given by the quasi-steady lift and drag forces (excluding added mass) determined from the lift and drag coefficients $C_L(\alpha)$ and $C_D(\alpha)$ as

$$\begin{aligned} f_{x,k} &= \tfrac{1}{2} \rho c U_k^2(z,\mathbf{x},\dot{\mathbf{x}}) \left(C_L(\alpha_k(z,\mathbf{x},\dot{\mathbf{x}})) \sin \varphi_k(z,\mathbf{x},\dot{\mathbf{x}}) - C_D(\alpha_k(z,\mathbf{x},\dot{\mathbf{x}})) \cos \varphi_k(z,\mathbf{x},\dot{\mathbf{x}}) \right) \\ f_{y,k} &= \tfrac{1}{2} \rho c U_k^2(z,\mathbf{x},\dot{\mathbf{x}}) \left(C_L(\alpha_k(z,\mathbf{x},\dot{\mathbf{x}})) \cos \varphi_k(z,\mathbf{x},\dot{\mathbf{x}}) + C_D(\alpha_k(z,\mathbf{x},\dot{\mathbf{x}})) \sin \varphi_k(z,\mathbf{x},\dot{\mathbf{x}}) \right) \end{aligned} \tag{15}$$

where U_k, φ_k, and α_k are the relative inflow velocity, inflow angle, and angle of attack, respectively. These variables are derived as (cf. Figure 8):

$$U_k = \sqrt{v_{x,ac,k}^2 + v_{y,ac,k}^2} \qquad \varphi_k = \arctan\left(\frac{v_{y,ac,k}}{v_{x,ac,k}}\right) \qquad \alpha_k = \arctan\left(\frac{v_{y,cp,k}}{v_{y,cp,k}}\right) \tag{16}$$

where $(v_{x,ac,k}, v_{y,ac,k})$ and $(v_{x,cp,k}, v_{y,cp,k})$ are the pairs of relative inflow velocities at the aerodynamic center and the collocation point[1] (cp), respectively, to the airfoil chord along blade k defined in its rotating frame of reference. These relative velocities can be derived as the x- and y-components of the vectors

$$\mathbf{v}_{ac,k} = (\mathbf{T}_{nac} \mathbf{T}_{DT} \mathbf{T}_{\psi_k})^{-1} \left(\dot{\mathbf{r}}_{ac,k} - \left\{ \begin{array}{c} 0 \\ W \\ 0 \end{array} \right\} \right) \quad \text{and} \quad \mathbf{v}_{cp,k} = (\mathbf{T}_{nac} \mathbf{T}_{DT} \mathbf{T}_{\psi_k})^{-1} \left(\dot{\mathbf{r}}_{cp,k} - \left\{ \begin{array}{c} 0 \\ W \\ 0 \end{array} \right\} \right) \tag{17}$$

where $\mathbf{r}_{cp,k}$ is the position vector of the collocation point in the ground fixed frame along blade k is

$$\mathbf{r}_{cp,k}(z,\mathbf{x}) = \left\{ \begin{array}{c} u_{tx} \\ u_{ty} \\ 0 \end{array} \right\} + \mathbf{T}_{nac} \left(\left\{ \begin{array}{c} 0 \\ L_s \\ 0 \end{array} \right\} + \mathbf{T}_{DT} \mathbf{T}_{\psi_k} \left(\left\{ \begin{array}{c} u_{x,k} \\ u_{y,k} \\ z \end{array} \right\} + \mathbf{T}_{\theta_k} \left\{ \begin{array}{c} -\frac{c}{2} \\ 0 \\ 0 \end{array} \right\} \right) \right) \tag{18}$$

where c is the chord length.

[1]The angle of attack is defined at the collocation point in the three-quarter chord where the boundary condition for a lumped-vortex representation of the lift would be evaluated [6]. This definition also captures the effects of torsional velocity $\dot{\theta}_k \neq 0$ on the aerodynamic forces leading to *pitch rate damping*.

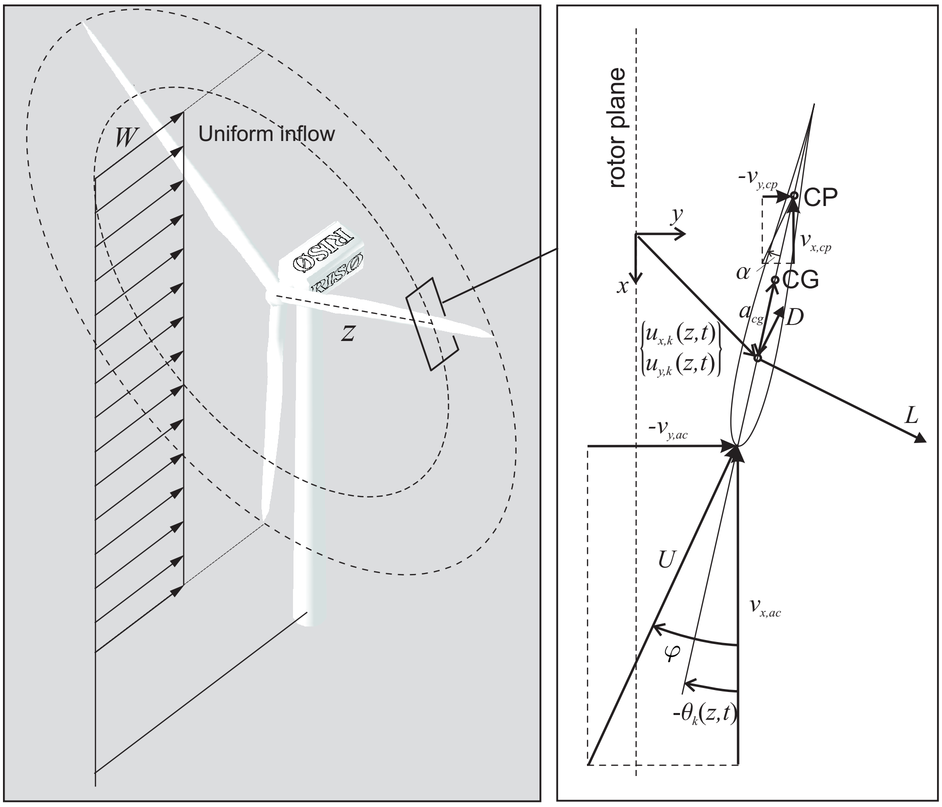


Figure 8: Schematics of a blade section at radius z showing the velocity triangle.

Insertion of (13) and (14) with (15 – 18) into (12) and linearization about zero steady state ($\mathbf{x} = \dot{\mathbf{x}} = \mathbf{0}$) yields the linear generalized aerodynamic forces

$$\mathbf{Q} = \left\{ \begin{array}{c} \mathbf{Q}_{0,b} \\ \mathbf{Q}_{0,b} \\ \mathbf{Q}_{0,b} \\ \mathbf{Q}_{0,t} \end{array} \right\} - \left[\begin{array}{cccc} \mathbf{C}_{a,b} & \mathbf{0} & \mathbf{0} & \mathbf{C}_{a,bt}(\psi_1) \\ \mathbf{0} & \mathbf{C}_{a,b} & \mathbf{0} & \mathbf{C}_{a,bt}(\psi_2) \\ \mathbf{0} & \mathbf{0} & \mathbf{C}_{a,b} & \mathbf{C}_{a,bt}(\psi_3) \\ \mathbf{C}_{a,tb}(\psi_1) & \mathbf{C}_{a,tb}(\psi_2) & \mathbf{C}_{a,tb}(\psi_3) & \mathbf{C}_{a,t} \end{array} \right] \dot{\mathbf{x}} - \left[\begin{array}{cccc} \mathbf{K}_{a,b} & \mathbf{0} & \mathbf{0} & \mathbf{K}_{a,bt}(\psi_1) \\ \mathbf{0} & \mathbf{K}_{a,b} & \mathbf{0} & \mathbf{K}_{a,bt}(\psi_2) \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_{a,b} & \mathbf{K}_{a,bt}(\psi_3) \\ \mathbf{K}_{a,tb}(\psi_1) & \mathbf{K}_{a,tb}(\psi_2) & \mathbf{K}_{a,tb}(\psi_3) & \mathbf{K}_{a,t} \end{array} \right] \mathbf{x} \tag{19}$$

where the steady state aerodynamic forces are

$$\mathbf{Q}_{0,b} = \tfrac{1}{2}\rho c \int_0^R U_0^2(z) \left\{ \begin{array}{c} C_{y0}(z)\,\phi_1(z) \\ C_{x0}(z)\,\phi_2(z) \\ 0 \end{array} \right\} dz \quad \text{and} \quad \mathbf{Q}_{0,b} = \tfrac{1}{2}\rho c \int_0^R U_0^2(z) \left\{ \begin{array}{c} 0 \\ 3C_{y0}(z) \\ 0 \\ 0 \\ 3C_{x0}(z)z \\ 3C_{x0}(z)z \end{array} \right\} dz \tag{20}$$

where $U_0 = \sqrt{W^2 + z^2\Omega^2}$ is the steady state inflow velocity, and $C_{x0} = C_L(\alpha_0)\sin\phi_0 - C_D(\alpha_0)\cos\phi_0$ and $C_{y0} = C_L(\alpha_0)\cos\phi_0 + C_D(\alpha_0)\sin\phi_0$ are tangential and thrust coefficients that are functions of the steady state angle of attack and inflow angle, that again are functions of radius: $\alpha_0 = \phi_0 = \arctan(W/(z\Omega))$.

The coupling matrices of the linear aerodynamic forces (12) can be written as

$$
\begin{aligned}
\mathbf{C}_{a,bt}(\psi_k) &= \mathbf{C}^{a_0}_{a,bt} + \mathbf{C}^{a_1}_{a,bt}\cos\psi_k + \mathbf{C}^{b_1}_{a,bt}\sin\psi_k \\
\mathbf{C}_{a,tb}(\psi_k) &= \mathbf{C}^{a_0}_{a,tb} + \mathbf{C}^{a_1}_{a,tb}\cos\psi_k + \mathbf{C}^{b_1}_{a,tb}\sin\psi_k \\
\mathbf{K}_{a,bt}(\psi_k) &= \mathbf{K}^{a_1}_{a,bt}\cos\psi_k + \mathbf{K}^{b_1}_{a,bt}\sin\psi_k \\
\mathbf{K}_{a,tb}(\psi_k) &= \mathbf{K}^{a_0}_{a,tb} + \mathbf{K}^{a_1}_{a,tb}\cos\psi_k + \mathbf{K}^{b_1}_{a,tb}\sin\psi_k
\end{aligned}
\tag{21}
$$

where the superscripts ^{a_0}, ^{a_1}, and ^{b_1} denote the rotor symmetric, cosine cyclic and sine cyclic components of the coupling matrices (a notation related to the Coleman transformation into multi-blade coordinates [3]). All aerodynamic system matrices are listed at the end of this note.

System matrices

The structural matrices for each blade are

$$
\mathbf{M}_b = m \begin{bmatrix} \int_0^R \phi_1^2 dz & 0 & -a_{cg} \int_0^R \phi_1 \phi_3 dz \\ 0 & \int_0^R \phi_2^2 dz & 0 \\ -a_{cg} \int_0^R \phi_1 \phi_3 dz & 0 & \int_0^R \phi_3^2 dz \end{bmatrix}
$$

$$
\mathbf{K}_b = \begin{bmatrix} \int_0^R \left(EI_x (\phi_1'')^2 + m\Omega^2 \frac{R^2-z^2}{2} (\phi_1')^2 \right) dz & 0 & 0 \\ 0 & \int_0^R \left(EI_y (\phi_2'')^2 + m\Omega^2 \left(\frac{R^2-z^2}{2} (\phi_2')^2 - \phi_2^2 \right) \right) dz & 0 \\ 0 & 0 & \int_0^R \left(GK (\phi_3')^2 + m\Omega^2 a_{cg}^2 \phi_3^2 \right) dz \end{bmatrix}, \tag{22}
$$

the structural matrices for nacelle/tower top coordinates are

$$
\mathbf{M}_t = \begin{bmatrix} 3mR+M & 0 & 0 & 3mRL_s & 0 & 0 \\ 0 & 3mR+M & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}mR(R^2+3{a_{cg}}^2) + 3mR{L_s}^2 + I_x & 0 & 0 & 0 \\ 3mRL_s & 0 & 0 & \frac{1}{2}mR(R^2+3{a_{cg}}^2) + 3mR{L_s}^2 + I_z & 0 & 0 \\ 0 & 0 & 0 & 0 & mR^3 + 3mR{a_{cg}}^2 + I_y & mR\left(R^2+3{a_{cg}}^2\right) \\ 0 & 0 & 0 & 0 & mR\left(R^2+3{a_{cg}}^2\right) & mR\left(R^2+3{a_{cg}}^2\right) \end{bmatrix}
$$

$$
\mathbf{G}_t = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -mR\Omega\left(R^2+3{a_{cg}}^2\right) & 0 & 0 \\ 0 & 0 & mR\Omega\left(R^2+3{a_{cg}}^2\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{K}_t = \begin{bmatrix} K_x & 0 & 0 & 0 & g_{xy} & 0 \\ 0 & K_y & -g_{xy} & 0 & 0 & 0 \\ 0 & -g_{xy} & G_x & 0 & 0 & 0 \\ 0 & 0 & 0 & G_z & 0 & 0 \\ g_{xy} & 0 & 0 & 0 & G_y & 0 \\ 0 & 0 & 0 & 0 & 0 & G_s \end{bmatrix}, \tag{23}
$$

and the structural coupling matrices are

$$\mathbf{M}_{tb} = \mathbf{M}_{bt}^{T} = m \begin{bmatrix} 0 & \cos\psi_k \int_0^R \phi_2 dz & 0 \\ \int_0^R \phi_1 dz & 0 & -a_{cg} \int_0^R \phi_3 dz \\ -\int_0^R \phi_1 \left(a_{cg}\sin\psi_k + z\cos\psi_k\right) dz & L_s \sin\psi_k \int_0^R \phi_2 dz & a_{cg} \int_0^R \phi_3 \left(a_{cg}\sin\psi_k + z\cos\psi_k\right) dz \\ -\int_0^R \phi_1 \left(a_{cg}\cos\psi_k - z\sin\psi_k\right) dz & L_s \cos\psi_k \int_0^R \phi_2 dz & a_{cg} \int_0^R \phi_3 \left(a_{cg}\cos\psi_k - z\sin\psi_k\right) dz \\ 0 & \int_0^R \phi_2 z dz & 0 \\ 0 & \int_0^R \phi_2 z dz & 0 \end{bmatrix}$$

$$\mathbf{G}_{bt} = m\Omega \begin{bmatrix} 0 & 0 & -2 \int_0^R \phi_1 \left(a_{cg}\cos\psi_k - z\sin\psi_k\right) dz & 2 \int_0^R \phi_1 \left(a_{cg}\sin\psi_k + z\cos\psi_k\right) dz & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\,a_{cg} \int_0^R \phi_2 dz & 2\,a_{cg} \int_0^R \phi_2 dz \\ 0 & 0 & 2\,a_{cg} \int_0^R \phi_3 \left(a_{cg}\cos\psi_k - z\sin\psi_k\right) dz & -2\,a_{cg} \int_0^R \phi_3 \left(a_{cg}\sin\psi_k + z\cos\psi_k\right) dz & 0 & 0 \end{bmatrix} \tag{24}$$

$$\mathbf{G}_{tb} = m\Omega \begin{bmatrix} 0 & -2\,\sin\psi_k \int_0^R \phi_2 dz & 0 \\ 0 & 0 & 0 \\ 0 & 2\,L_s \cos\psi_k \int_0^R \phi_2 dz & 0 \\ 0 & -2\,L_s \sin\psi_k \int_0^R \phi_2 dz & 0 \\ 0 & -2\,a_{cg} \int_0^R \phi_2 dz & 0 \\ 0 & -2\,a_{cg} \int_0^R \phi_2 dz & 0 \end{bmatrix}$$

$$\mathbf{K}_{tb} = m\Omega^2 \begin{bmatrix} 0 & -\cos\psi_k \int_0^R \phi_2 dz & 0 \\ 0 & 0 & 0 \\ -\int_0^R \phi_1 \left(a_{cg}\sin\psi_k + z\cos\psi_k\right) dz & -L_s \sin\psi_k \int_0^R \phi_2 dz & a_{cg} \int_0^R \phi_3 \left(a_{cg}\sin\psi_k + z\cos\psi_k\right) dz \\ -\int_0^R \phi_1 \left(a_{cg}\cos\psi_k - z\sin\psi_k\right) dz & -L_s \cos\psi_k \int_0^R \phi_2 dz & a_{cg} \int_0^R \phi_3 \left(a_{cg}\cos\psi_k - z\sin\psi_k\right) dz \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The aerodynamic matrices for each blade are ($\lambda = z\Omega/W$, $C_{x0} = C_L(\alpha_0)\sin\phi_0 - C_D(\alpha_0)\cos\phi_0$, and $C_{y0} = C_L(\alpha_0)\cos\phi_0 + C_D(\alpha_0)\sin\phi_0$)

$$\mathbf{C}_{a,b} = \tfrac{1}{4}c\rho W \int_0^R \begin{bmatrix} 4\,(\phi_1)^2\,C_{y0} + 2\,(\phi_1)^2\,C'_{y0}\,\lambda - 2\,(\phi_1)^2\,C_{x0}\,\lambda & 2\,\phi_1\phi_2 C'_{y0} - 2\,\phi_1\phi_2 C_{x0} - 4\,\phi_1\phi_2 C_{y0}\,\lambda & -c\phi_1\phi_3 C'_{y0}\,\lambda \\ 4\,\phi_1\phi_2 C_{x0} + 2\,\phi_1\phi_2 C'_{x0}\,\lambda + 2\,\phi_1\phi_2 C_{y0}\,\lambda & 2\,(\phi_2)^2\,C'_{x0} + 2\,(\phi_2)^2\,C_{y0} - 4\,(\phi_2)^2\,C_{x0}\,\lambda & -c\phi_2\phi_3 C'_{x0}\,\lambda \\ 0 & 0 & 0 \end{bmatrix} dz$$

$$\mathbf{K}_{a,b} = \tfrac{1}{2}c\rho \int_0^R U_0^2 \begin{bmatrix} 0 & 0 & -\phi_1\phi_3 C'_{y0} \\ 0 & 0 & -\phi_2\phi_3 C'_{x0} \\ 0 & 0 & 0 \end{bmatrix} dz, \tag{25}$$

the aerodynamic matrices for nacelle/tower top coordinates are

$$\mathbf{C}_{a,t} = \tfrac{1}{8}c\rho W \int_0^R \left[\begin{matrix} 6C'_{x0} + 6C_{y0} - 12C_{x0}\lambda & 0 & -6\lambda z C'_{x0} - 6\lambda z C_{y0} - 12zC_{x0} \\ 0 & 24C_{y0} + 12C'_{y0}\lambda - 12C_{x0}\lambda & 0 \\ -6zC'_{y0} + 12\lambda z C_{y0} + 6zC_{x0} & 0 & 6z^2C'_{y0}\lambda + \left(6{L_s}^2 - 3L_s c\lambda\right)C'_{x0} + \left(12z^2 + 6{L_s}^2\right)C_{y0} + \left(-6z^2\lambda - 12{L_s}^2\lambda\right)C_{x0} \\ 6L_sC'_{x0} + 6L_sC_{y0} - 12L_sC_{x0}\lambda & 0 & -3\left(-2L_s + c\lambda\right)zC'_{y0} - 6L_s\lambda zC'_{x0} - 18L_s\lambda zC_{y0} - 18zL_sC_{x0} \\ 0 & 12\lambda zC'_{x0} + 12\lambda zC_{y0} + 24zC_{x0} & 0 \\ 0 & 12\lambda zC'_{x0} + 12\lambda zC_{y0} + 24zC_{x0} & 0 \end{matrix} \right.$$

$$\left. \begin{matrix} \left(6L_s - 3c\lambda\right)C'_{x0} + 6L_sC_{y0} - 12L_sC_{x0}\lambda & 0 & 0 \\ 0 & 12zC'_{y0} - 24\lambda zC_{y0} - 12zC_{x0} & 12zC'_{y0} - 24\lambda zC_{y0} - 12zC_{x0} \\ 3\left(-2L_s + c\lambda\right)zC'_{y0} + 6L_s\lambda zC'_{x0} + 18L_s\lambda zC_{y0} + 18zL_sC_{x0} & 0 & 0 \\ 6z^2C'_{y0}\lambda + \left(6{L_s}^2 - 3L_sc\lambda\right)C'_{x0} + \left(12z^2 + 6{L_s}^2\right)C_{y0} + \left(-6z^2\lambda - 12{L_s}^2\lambda\right)C_{x0} & 0 & 0 \\ 0 & 12z^2C'_{x0} + 12z^2C_{y0} - 24z^2C_{x0}\lambda & 12z^2C'_{x0} + 12z^2C_{y0} - 24z^2C_{x0}\lambda \\ 0 & 12z^2C'_{x0} + 12z^2C_{y0} - 24z^2C_{x0}\lambda & 12z^2C'_{x0} + 12z^2C_{y0} - 24z^2C_{x0}\lambda \end{matrix} \right] dz$$

$$\mathbf{K}_{a,t} = \tfrac{1}{4}c\rho W^2 \int_0^R \begin{bmatrix} 0 & 0 & 0 & 3C_{y0} + 6C_{y0}\lambda^2 + 6C_{x0}\lambda - 3C'_{x0} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3L_sC'_{x0} - 3L_sC_{y0} + 6L_sC_{x0}\lambda & \left(6C_{x0}\lambda^2 - 6C_{y0}\lambda + 3C_{x0} + 3C'_{y0}\right)z & 0 & 0 \\ 0 & 0 & \left(-3C'_{y0} + 3C_{x0} + 6C_{y0}\lambda\right)z & -3L_sC'_{x0} - 3L_sC_{y0} + 6L_sC_{x0}\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} dz \tag{26}$$

the symmetric and cyclic components of the aerodynamic coupling matrices from nacelle/tower top coordinates to blade coordinates are

$$
\begin{aligned}
\mathbf{C}_{a,bt}^{a_0} &= \tfrac{1}{2}c\rho W \int_0^R \begin{bmatrix} 0 & 2\,\phi_1 C_{y0} + \phi_1 C'_{y0}\,\lambda - \phi_1 C_{x0}\,\lambda & 0 & 0 & -\phi_1\left(-C'_{y0} + C_{x0} + 2\,C_{y0}\,\lambda\right) z & -\phi_1\left(-C'_{y0} + C_{x0} + 2\,C_{y0}\,\lambda\right) z \\ 0 & 2\,\phi_2 C_{x0} + \phi_2 C'_{x0}\,\lambda + \phi_2 C_{y0}\,\lambda & 0 & 0 & -\phi_2\left(-C'_{x0} - C_{y0} + 2\,C_{x0}\,\lambda\right) z & -\phi_2\left(-C'_{x0} - C_{y0} + 2\,C_{x0}\,\lambda\right) z \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} dz \\
\mathbf{C}_{a,bt}^{a_1} &= \tfrac{1}{2}c\rho W \int_0^R \begin{bmatrix} \phi_1 C'_{y0} - \phi_1 C_{x0} - 2\,\phi_1 C_{y0}\,\lambda & 0 & -\phi_1\left(2\,C_{y0} + C'_{y0}\,\lambda - C_{x0}\,\lambda\right) z & \phi_1 L_s\,C'_{y0} - 2\,\phi_1 L_s\,C_{y0}\,\lambda - \phi_1 L_s\,C_{x0} - 1/2\,\phi_1 c C'_{y0}\,\lambda & 0 & 0 \\ \phi_2 C_{y0} - 2\,\phi_2 C_{x0}\,\lambda + \phi_2 C'_{x0} & 0 & -\phi_2\left(2\,C_{x0} + C'_{x0}\,\lambda + C_{y0}\,\lambda\right) z & -2\,\phi_2 L_s\,C_{x0}\,\lambda - 1/2\,\phi_2 c C'_{x0}\,\lambda + \phi_2 L_s\,C_{y0} + \phi_2 L_s\,C'_{x0} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} dz \\
\mathbf{C}_{a,bt}^{b_1} &= \tfrac{1}{2}c\rho W \int_0^R \begin{bmatrix} 0 & 0 & \phi_1 L_s\,C'_{y0} - 2\,\phi_1 L_s\,C_{y0}\,\lambda - \phi_1 L_s\,C_{x0} - 1/2\,\phi_1 c C'_{y0}\,\lambda & \phi_1\left(2\,C_{y0} + C'_{y0}\,\lambda - C_{x0}\,\lambda\right) z & 0 & 0 \\ 0 & 0 & -2\,\phi_2 L_s\,C_{x0}\,\lambda - 1/2\,\phi_2 c C'_{x0}\,\lambda + \phi_2 L_s\,C_{y0} + \phi_2 L_s\,C'_{x0} & \phi_2\left(2\,C_{x0} + C'_{x0}\,\lambda + C_{y0}\,\lambda\right) z & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} dz \qquad (27) \\
\mathbf{K}_{a,bt}^{a_1} &= \tfrac{1}{2}c\rho W^2 \int_0^R \begin{bmatrix} 0 & 0 & 0 & -\phi_1 C'_{y0} + \phi_1 C_{x0} + 2\,\phi_1 C_{y0}\,\lambda & 0 & 0 \\ 0 & 0 & 0 & -\phi_2 C'_{x0} - \phi_2 C_{y0} + 2\,\phi_2 C_{x0}\,\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} dz \\
\mathbf{K}_{a,bt}^{b_1} &= \tfrac{1}{2}c\rho W^2 \int_0^R \begin{bmatrix} 0 & 0 & -\phi_1 C'_{y0} + \phi_1 C_{x0} + 2\,\phi_1 C_{y0}\,\lambda & 0 & 0 & 0 \\ 0 & 0 & -\phi_2 C'_{x0} - \phi_2 C_{y0} + 2\,\phi_2 C_{x0}\,\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} dz,
\end{aligned}
$$

the symmetric and cyclic components of the aerodynamic coupling matrices from nacelle/tower top coordinates to blade coordinates are

$$
\begin{aligned}
\mathbf{C}^{a_0}_{a,tb} &= \tfrac{1}{2} c\rho W \int_0^R \begin{bmatrix} 0 & 0 & 0 \\ 2\,\phi_1 C_{y0} + \phi_1 C'_{y0}\,\lambda - \phi_1 C_{x0}\,\lambda & \phi_2 C'_{y0} - 2\,\phi_2 C_{y0}\,\lambda - \phi_2 C_{x0} & -1/2\, c\phi_3 C'_{y0}\,\lambda \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \phi_1\,(2\,C_{x0} + C'_{x0}\,\lambda + C_{y0}\,\lambda)\,z & -\phi_2\,(-C'_{x0} - C_{y0} + 2\,C_{x0}\,\lambda)\,z & -1/2\, c\phi_3 C'_{x0}\,\lambda\, z \\ \phi_1\,(2\,C_{x0} + C'_{x0}\,\lambda + C_{y0}\,\lambda)\,z & -\phi_2\,(-C'_{x0} - C_{y0} + 2\,C_{x0}\,\lambda)\,z & -1/2\, c\phi_3 C'_{x0}\,\lambda\, z \end{bmatrix} dz \\
\mathbf{C}^{a_1}_{a,tb} &= \tfrac{1}{2} c\rho W \int_0^R \begin{bmatrix} 2\,\phi_1 C_{x0} + \phi_1 C'_{x0}\,\lambda + \phi_1 C_{y0}\,\lambda & \phi_2 C_{y0} - 2\,\phi_2 C_{x0}\,\lambda + \phi_2 C'_{x0} & -1/2\, c\phi_3 C'_{x0}\,\lambda \\ 0 & 0 & 0 \\ -\phi_1 \left(2\,C_{y0} + C'_{y0}\,\lambda - C_{x0}\,\lambda\right) z & \phi_2 \left(-C'_{y0} + C_{x0} + 2\,C_{y0}\,\lambda\right) z & 1/2\, c\phi_3 C'_{y0}\,\lambda\, z \\ 2\,\phi_1 L_s\, C_{x0} + \phi_1 L_s\, C'_{x0}\,\lambda + \phi_1 L_s\, C_{y0}\,\lambda & \phi_2 L_s\, C_{y0} - 2\,\phi_2 L_s\, C_{x0}\,\lambda + \phi_2 L_s\, C'_{x0} & -1/2\, c\phi_3 L_s\, C'_{x0}\,\lambda \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} dz \\
\mathbf{C}^{b_1}_{a,tb} &= \tfrac{1}{2} c\rho W \int_0^R \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2\,\phi_1 L_s\, C_{x0} + \phi_1 L_s\, C'_{x0}\,\lambda + \phi_1 L_s\, C_{y0}\,\lambda & \phi_2 L_s\, C_{y0} - 2\,\phi_2 L_s\, C_{x0}\,\lambda + \phi_2 L_s\, C'_{x0} & -1/2\, c\phi_3 L_s\, C'_{x0}\,\lambda \\ \phi_1 \left(2\,C_{y0} + C'_{y0}\,\lambda - C_{x0}\,\lambda\right) z & -\phi_2 \left(-C'_{y0} + C_{x0} + 2\,C_{y0}\,\lambda\right) z & -1/2\, c\phi_3 C'_{y0}\,\lambda\, z \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} dz \qquad (28) \\
\mathbf{K}^{a_0}_{a,tb} &= \tfrac{1}{2} c\rho \int_0^R U_0^2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\phi_3 C'_{y0} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\phi_3 C'_{x0}\, z \\ 0 & 0 & -\phi_3 C'_{x0}\, z \end{bmatrix} dz \\
\mathbf{K}^{a_1}_{a,tb} &= \tfrac{1}{2} c\rho \int_0^R U_0^2 \begin{bmatrix} 0 & 0 & -\phi_3 C'_{x0} \\ 0 & 0 & 0 \\ 0 & 0 & \phi_3 z C'_{y0} \\ \phi_1 C_{x0} & -\phi_2 C_{y0} & -\phi_3 L_s\, C'_{x0} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} dz \\
\mathbf{K}^{b_1}_{a,tb} &= \tfrac{1}{2} c\rho \int_0^R U_0^2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \phi_1 C_{x0} & -\phi_2 C_{y0} & -\phi_3 L_s\, C'_{x0} \\ 0 & 0 & -\phi_3 z C'_{y0} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} dz
\end{aligned}
$$

Appendix D – Pre-test slides (16 pages)

Pre-test in Wind Turbine Dynamics and Aeroelasticity

The following slides contain 14 multiple-choice questions.

You will have 1 minute to answer each question.

Please cross-out your answer on your answer-sheet.

Please cross-out only ONE answer.

Please remember that your evaluation
will NOT depend on this test.

Question 1:

What name is related to this degree of freedom on a wind turbine shown on the figure?

A	B	C	D	E
Azimuth	Yaw	Tilt	Pitch	No idea, or question not understood

Question 2:

At which wind speeds do the blades of a **stall-regulated** wind turbine with an asynchronous generator deliberately operate in stall (flow separation)?

A	B	C	D	E
Never	**At low wind speeds**	**Around the rated wind speed**	**At high wind speeds**	**No idea, or question not understood**

Question 3:

At which wind speeds do the blades of a **pitch-regulated** wind turbine with a variable speed generator deliberately operate in stall?

A	B	C	D	E
Never	**At low wind speeds**	**Around the rated wind speed**	**At high wind speeds**	**No idea, or question not understood**

Question 4:

What is the normal operation range of the Tip Speed Ratio (tangential tip velocity due to rotor rotation over free-stream wind velocity) for a MW-sized three-bladed wind turbine?

A	B	C	D	E
0.1 – 5	5 – 15	15 – 30	30 – 50	No idea, or question not understood

Question 5:

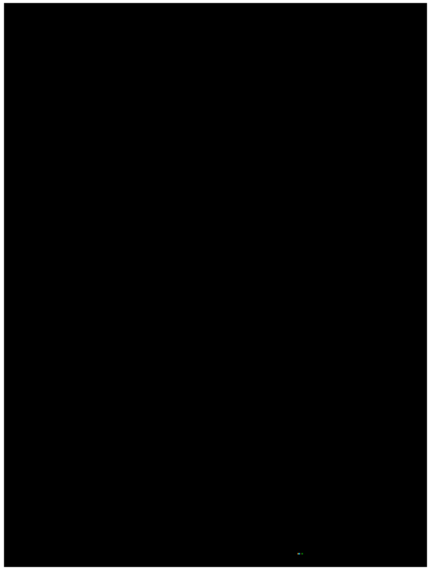
Is the rated rotor speed of a typical onshore MW-sized three-bladed wind turbine higher, or lower than its lowest natural frequency?

A	B	C	D	E
Lower	**Higher**	**-**	**-**	**No idea, or question not understood**

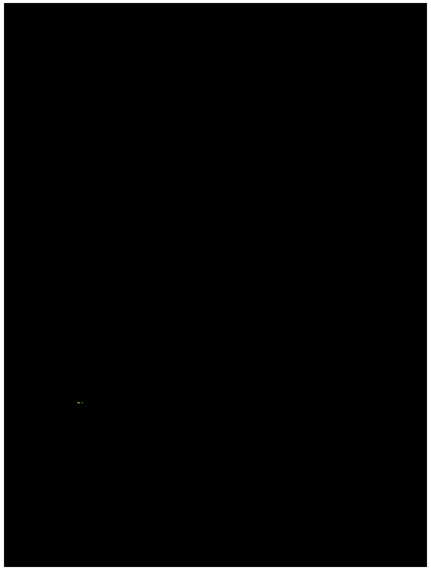
Question 6:

Which of these modes is the first yaw mode?

1



2



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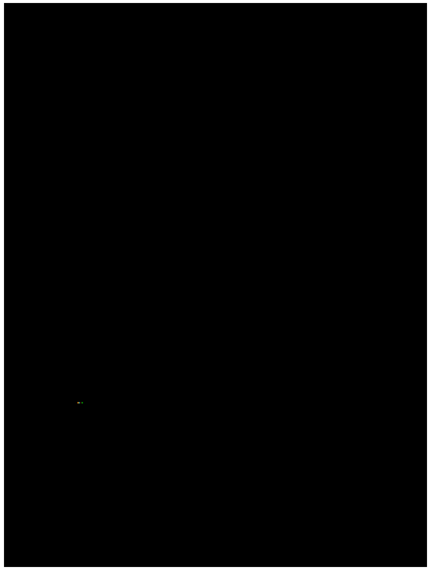

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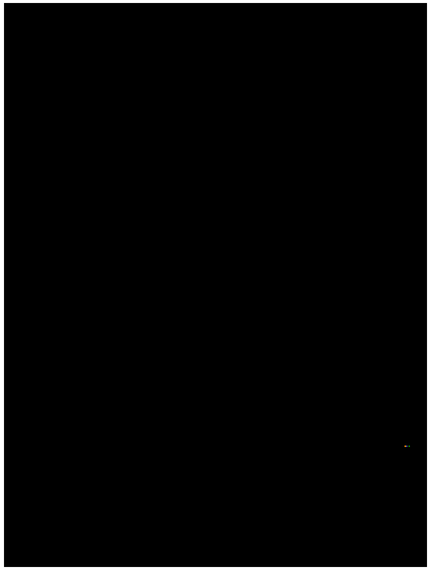
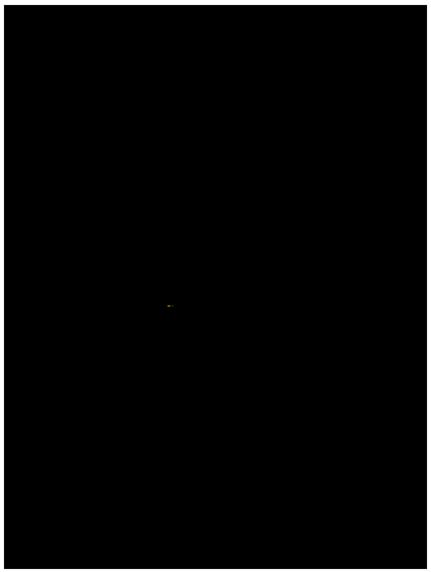
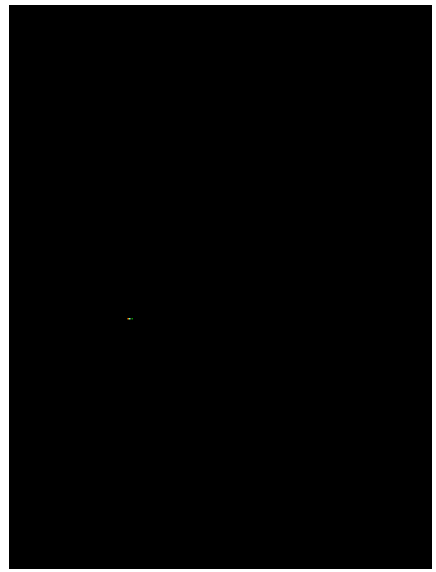


A	B	C	D	E
1	2	3	4	No idea, or question not understood

Question 7:

Order these modes of a turbine at standstill according to their natural frequency, begin with the lowest:


1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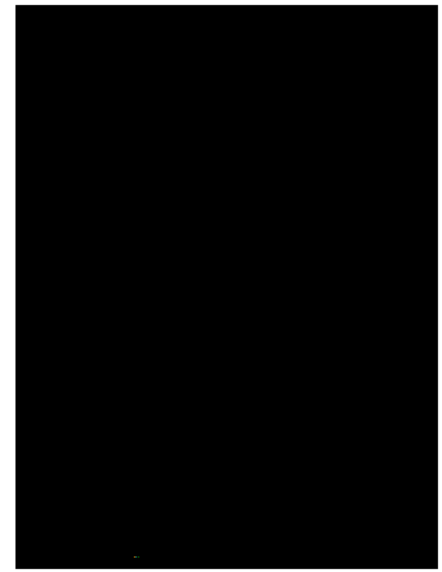

3

A	B	C	D	E
1 – 2 – 3	**2 – 3 – 1**	**3 – 2 – 1**	**2 – 1 – 3**	**No idea, or question not understood**

Question 8:

A pair of asymmetric rotor modes of a three-bladed turbine related to a blade mode with frequency f may couple and become two **pure** backward whirling (BW) and forward whirling (FW) modes during operation. What is the frequency of these modes when measured on a blade?

A	B	C	D	E
BW = f FW = f	BW = $f + \Omega$ FW = $f - \Omega$	BW = $f - \Omega$ FW = $f + \Omega$	-	No idea, or question not understood

Question 9:

Which of these parameters is **least** important for the risk of **stall-induced vibrations** for a three-bladed turbine?

A	B	C	D	E
Direction of blade vibrations relative to rotor plane	Angle of attack	Torsional blade stiffness	Structural damping	No idea, or question not understood

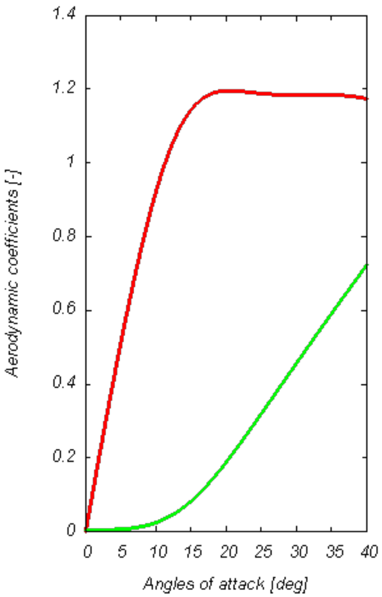
Question 10:

Which of these parameters is **least** important for the risk of **classical flutter** for a three-bladed turbine?

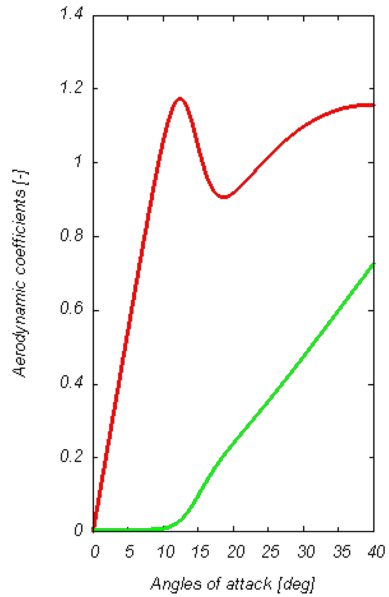
A	B	C	D	E
Rotor speed	Angle of attack	Torsional blade stiffness	Structural damping	No idea, or question not understood

Question 11:

Based on their characteristics; which airfoil has the highest risk of stall-induced vibrations?



1



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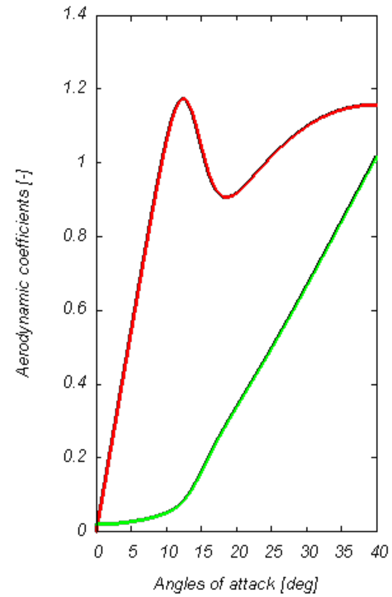
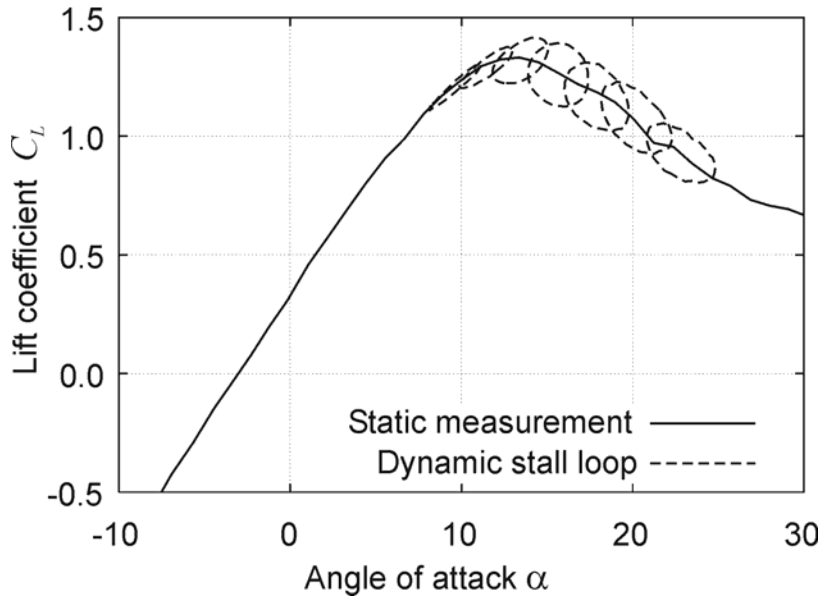


3

A	B	C	D	E
1	2	3	-	No idea, or question not understood

Question 12:

What is the effect of dynamic stall (unsteady behavior of lift and drag in stall) on the aerodynamic damping of turbine modes?



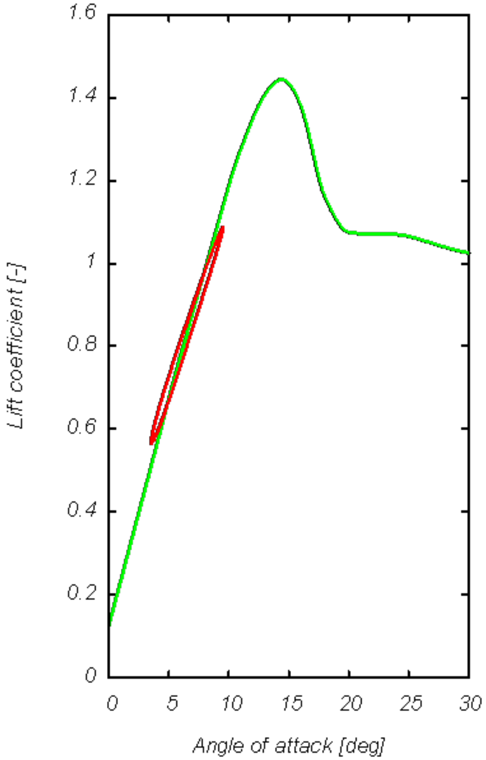
A	B	C	D	E
None	Stabilizing	Destabilizing	-	No idea, or question not understood

DTU

RISØ

Question 13:

What is the effect of shed vorticity (unsteady behavior of lift and drag in attached flow given by the Theodorsen theory) on the risk of classical flutter, where flapwise bending and torsion of the blade becomes negative damped?



A	B	C	D	E
None	Stabilizing	Destabilizing	-	No idea, or question not understood

Question 14:

Based on position of the center of gravity (CG) relative to the aerodynamic center (AC) and elastic axis (EA); which airfoil section has **no** risk of classical flutter?

1

2

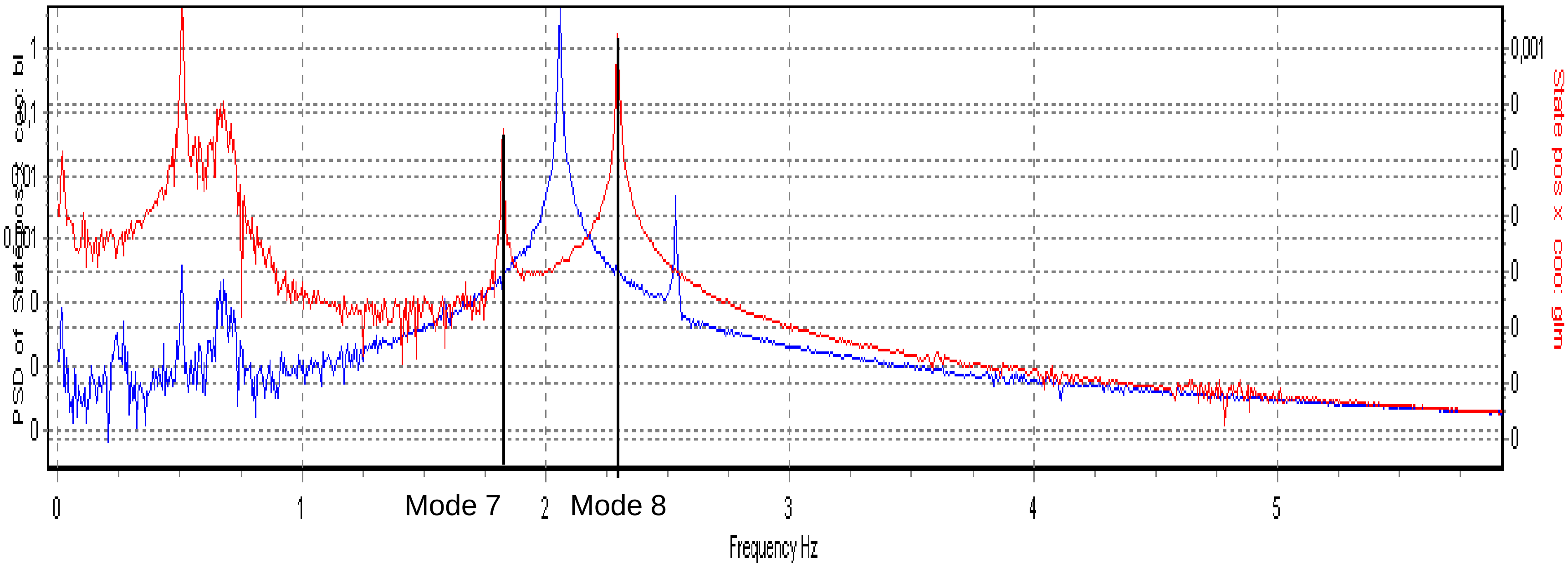
3

A	B	C	D	E
1	2	3	-	No idea, or question not understood

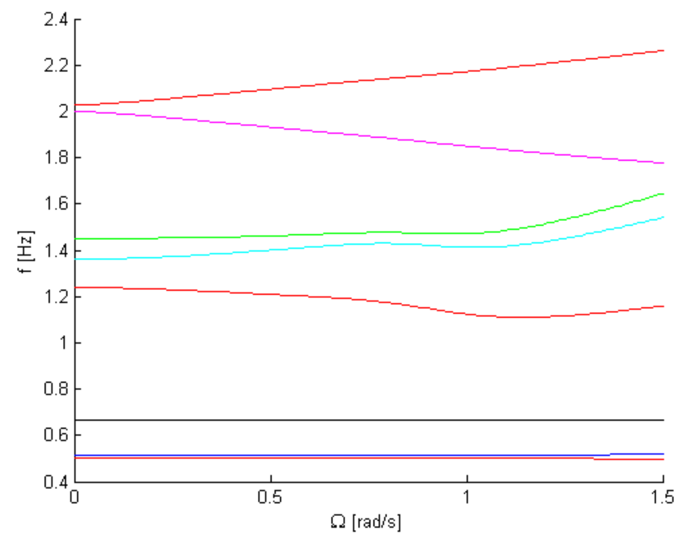
ANSWER SHEET – WT DYNAMICS & AEROELASTICITY

	A	B	C	D	E
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
13					
14					

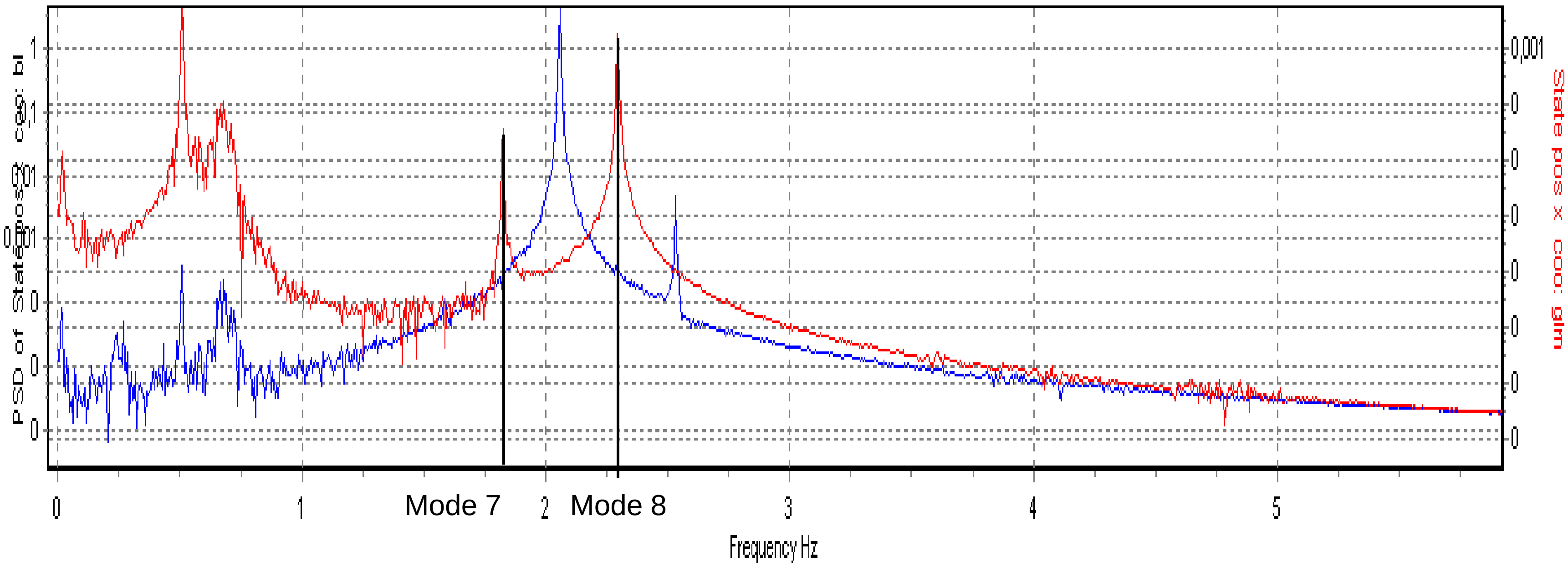
Appendix E – A student presentation (3 pages)



Wind speed = 12 m/s
Omega = 1.5 rad/s (0.24 Hz)

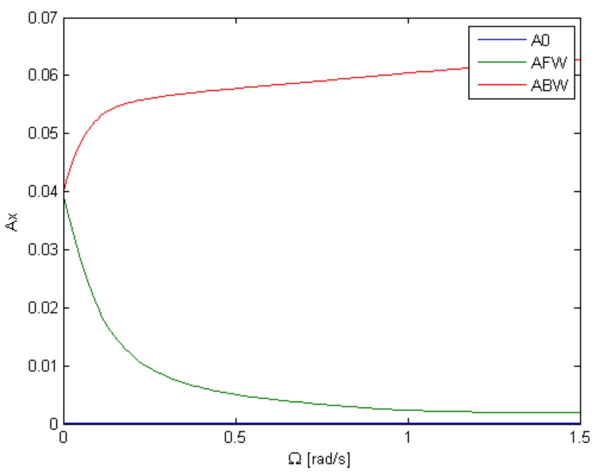


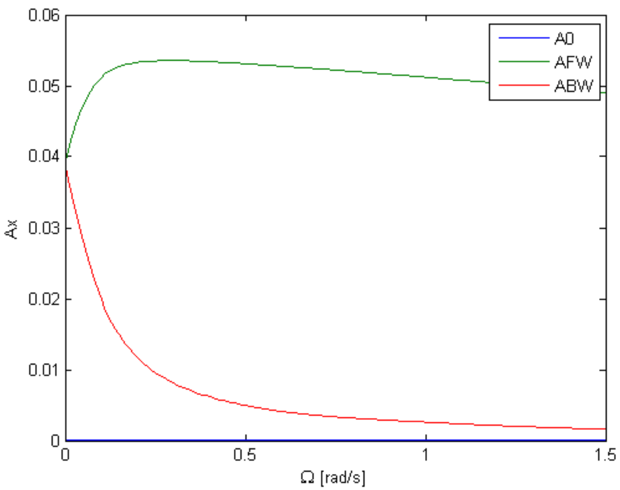
Blue = Blade edge (in blade coor)
Red = Tower top side-side (fixed frame)



Mode 7: edge amp.

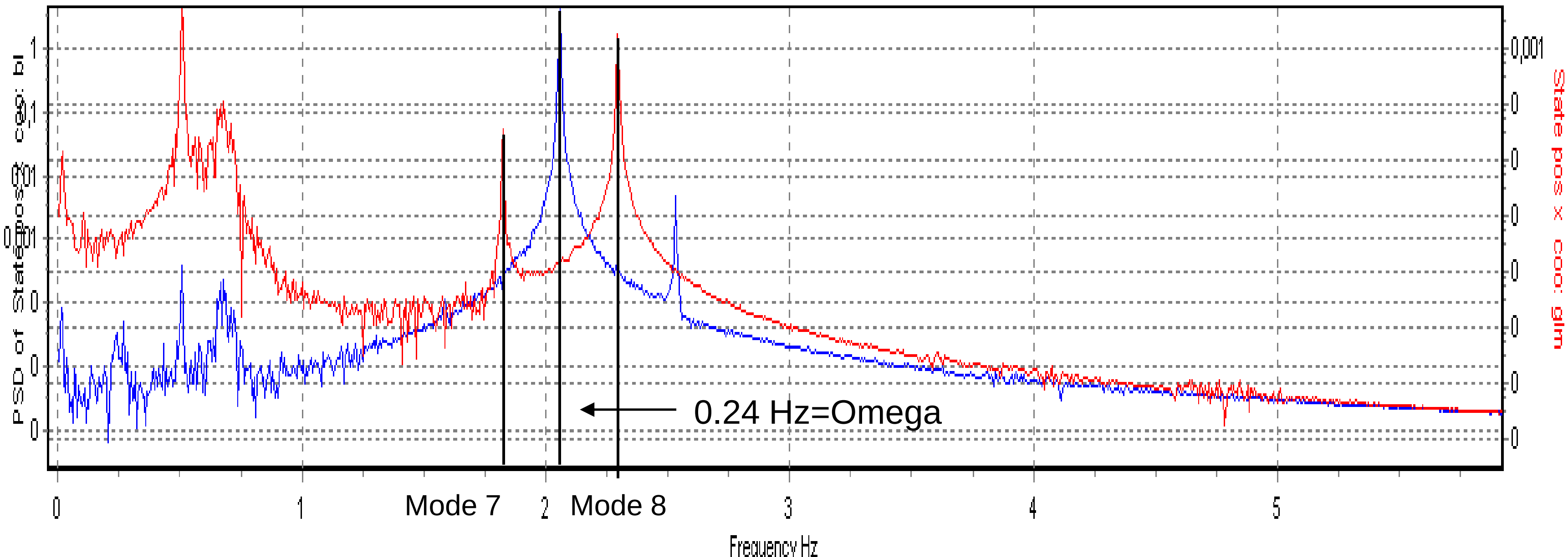
Mode 8: edge amp.



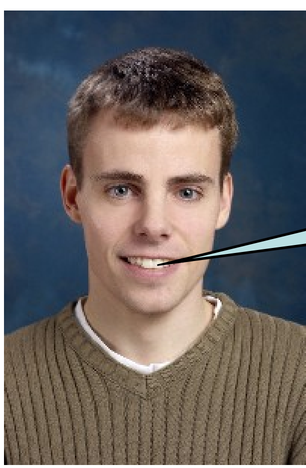


Edge backward whirling

Edge foreward whirling



$$
\begin{aligned}
q_k = A_0 \sin\big(\omega t + \phi_0\big) \quad &+ \quad A_{\mathrm{BW}} \sin\Big((\omega + \Omega)t + \tfrac{2\pi}{3}(k-1) + \phi_{\mathrm{BW}}\Big) \\
&+ \quad A_{\mathrm{FW}} \sin\Big((\omega - \Omega)t - \tfrac{2\pi}{3}(k-1) + \phi_{\mathrm{FW}}\Big)
\end{aligned}
$$



Appendix F – DCAMM evaluation sheet (3 pages)

Course Evaluation

Course: **Wind Turbine Dynamics and Aeroelasticity**

Date: **June 23rd-27th, 2008**

We need your assistance for improving future versions of this course. We would appreciate your taking the time to complete the following:

1) Name (optional) ____

2) Level: Master student ☐ PhD-student ☐ PhD ☐ Other ☐

3) Would you recommend this course to others? Yes ☐ No ☐ Perhaps ☐

4) Did the course match your *expectations* regarding

Content:

____

Level:

____

Work load:

____

5) What is your opinion on the suitability of this School?

excellent	
very good	
good	
moderate	
poor	

6) How many days did you follow the total working days of the school?

7) Did you follow the exercises?

8) What is your general opinion on the presentation of the lectures?

excellent	
very good	
good	
moderate	
poor	

9) What is your general opinion on the technical value of the printed materials?

excellent	
very good	
good	
moderate	
poor	

10) What was *good* and should be kept in future versions of the course?

11) What was *less good* and could be changed or improved on?

12) Do you have any suggestions for changing the *daily schedule* of the course

13) What is your opinion of the accommodation?

excellent	
very good	
good	
moderate	
poor	

14) Any other comments or suggestions?

15) What items did you wish to learn but were not covered?

16) Overall, how much do you feel you have learned from this course?

an exceptional amount	
a great deal	
a considerable amount	
a modest amount	